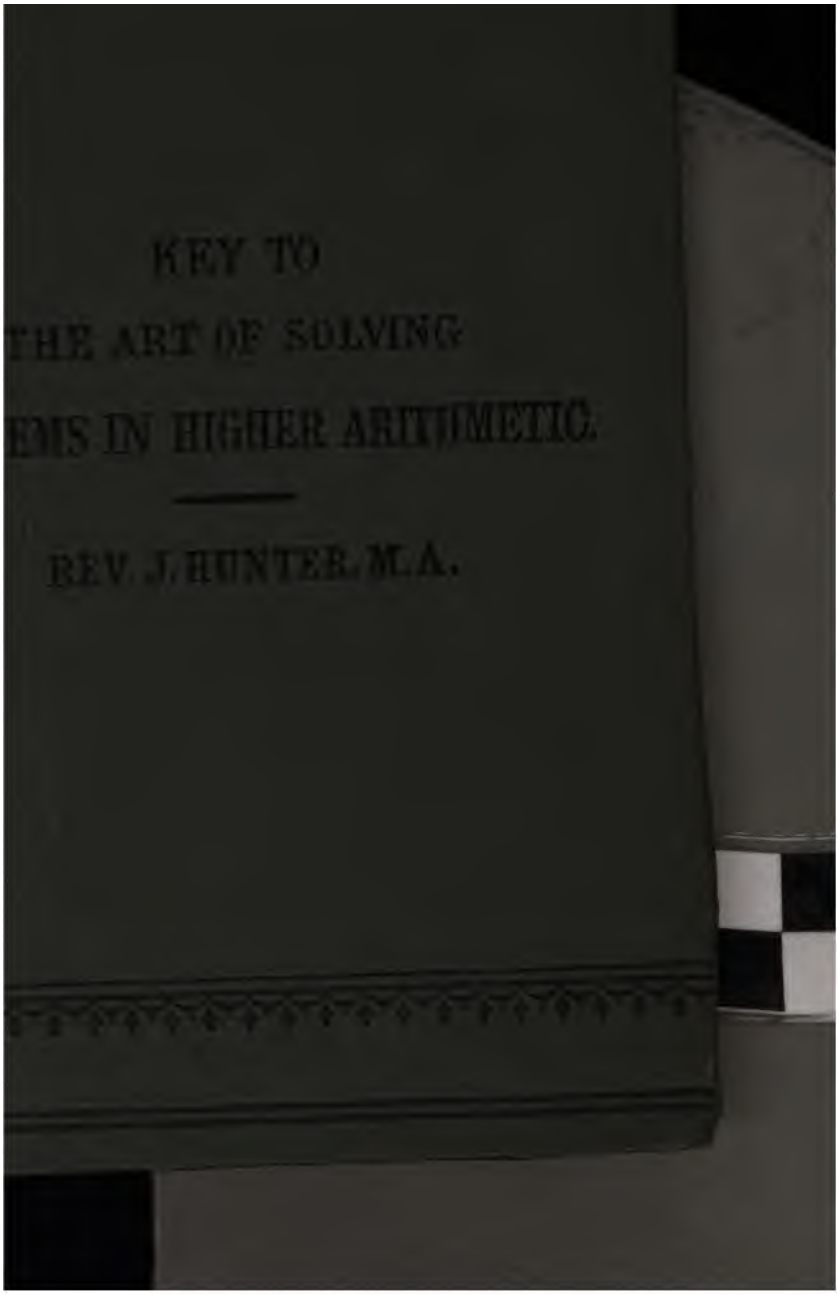
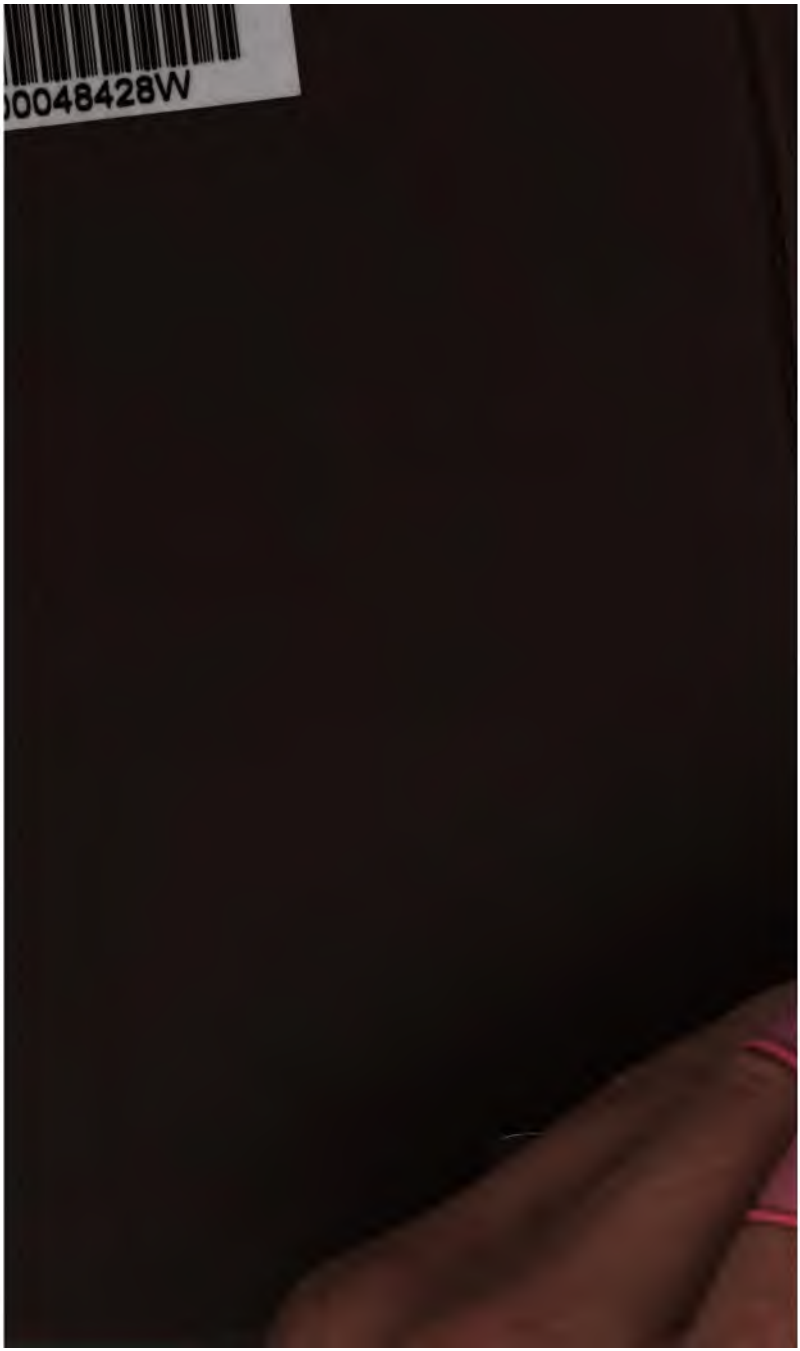

KEY TO
THE ART OF SOLVING
EMS IN HIGHER ARITHMETIC.
REV. J. HUNTER, M.A.



KEY

TO

PROBLEMS IN HIGHER ARITHMETIC

*** On page 63 of *The Art of Solving Problems &c.* the solution of Ex. 13 should be as follows :—

Sold out for £3640, less £$\frac{1}{2}$ for each cent. £$78\frac{3}{4}$ is income for $13\frac{1}{8}$ cents. at 92, = £$1207\frac{1}{2}$; ∴ an investment of £$2432\frac{1}{2}$, less £$\frac{1}{2}$ for each cent., would again obtain £$4\frac{1}{2}$ income for each cent., and cost £$4\frac{1}{2} \div \frac{6}{92}$, or £69 for each cent., and hence £69 for each cent. is £$2432\frac{1}{2}$, less £$\frac{1}{2}$ for each cent. ∴ $2432\frac{1}{2} \div 69\frac{1}{2} = 35$ cents. at $3640 \div 35$, or 104. *Ans.*

KEY

TO

THE ART OF SOLVING

PROBLEMS IN HIGHER ARITHMETIC

BY THE

REV. J. HUNTER, M.A.

LONDON
LONGMANS, GREEN, AND CO.
1884

LONDON : PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
AND PARLIAMENT STREET

KEY

TO

HUNTER'S ART OF SOLVING PROBLEMS IN HIGHER ARITHMETIC.

EXERCISES 1.

1. $\frac{1}{2}$ (£13. 10*s*. $7\frac{1}{2}d$. + 13*s*. $10\frac{1}{2}d$.) = £7. 2*s*. 3*d*.
   £6. 8*s*. $4\frac{1}{2}d$ } *Ans.*

2. $\frac{1}{2}$ (£72. 2*s*. 6*d*. − £23. 2*s*. 6*d*.) = £24. 12*s*. 6*d*. *Ans.*

3. $\frac{1}{2}$ (583ac. 1r. 37p. + 67ac. 2r. 33p.) = 325 ac. 2 r. 15 p.
   257 3 22 } *Ans.*

4. $\frac{1}{2}$ $(234\frac{15}{18} + 67\frac{16}{18}) = 302\frac{13}{18} \div 2 = 151\frac{13}{36}$
   $83\frac{17}{36}$ } *Ans.*

5. $\frac{1}{2}$ of $\frac{49+34}{56} = \frac{83}{112}$
   $\frac{98-83}{112} = \frac{15}{112}$ } *Ans.*

6. Sum of rates 74
   Difference 5
   2)79
   Ans. $34\frac{1}{2}$ and $39\frac{1}{2}$ mi.

7.

$\cdot 18\frac{7}{9} = \frac{1}{100}$ of $\frac{162}{9}$;
sq. root $= \frac{1}{10}$ of $\frac{13}{3}$;
$\frac{1}{2}(5\frac{5}{30} + \frac{13}{30}) = 2\frac{4}{5}$
$2\frac{24}{30} - \frac{13}{30} = 2\frac{11}{30}$ } *Ans.*

8.

Length + breadth, 1300
Length − breadth, 348
2)952
Ans. 824 and 476 yds.

9. $\frac{28-15}{36}=\frac{13}{36}$ | $\frac{1}{2}$ of $\frac{65-42}{180}=\frac{23}{360}$

$\frac{3+4}{30}=\frac{7}{30}$ | $\frac{130-23}{360}=\frac{107}{360}$ } *Ans.*

10. $AB+BC+AC=AC+123+AC=235$;

$\therefore AC=\frac{1}{2}(235-123)=56$;

thus we have $BC+AB=56+123$, and $BC-AB=7$;

$\frac{1}{2}(179+7)$ and $\frac{1}{2}(179-7)$ are 93 and 86;

hence AB 86, BC 93, AC 56. *Ans.*

11.

£456. 10*s*. is the sum of A's investment and A's gain;

£403. 10*s*. equals the difference of these quantities.

£860. 0*s*. $\div 2$ = £430. *Ans.*

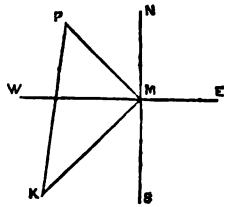
12.

Difference of the diameters $=333\div 3{\cdot}1416=106$ nearly.

Sum of ditto $=134\times 2=268$.

Hence the halves of 374 and 162 = 187 and 81. *Ans.*

EXERCISES 2.



1. Let P, M, and K mark the relative situations of the three places; the angle PMK is a right angle; its containing sides PM = 400 and KM = 500;

$\therefore PK = 100\sqrt{16+25}$

= about 640 miles. *Ans.*

2. The sum of length and breadth is 553, and their difference 263, $\therefore$ length

408, breadth 145; and hence the diagonal, being the hypotenuse of a right-angled triangle, $= \sqrt{408^2+145^2}$ $= \sqrt{187489} = 433$. *Ans.*

3. AC 15 ft., ACB the right angle; $AC^2 = 225$; hence, $225 \div 4\frac{1}{6} = 54$ ft. $= AB + BC$,

$$4\frac{1}{6} = AB - BC$$

$$49\frac{5}{6} \div 2 = \frac{299}{12} = BC;$$

$\frac{1}{2}$ BC $\times$ 15 $= 186\frac{7}{8}$ sq. ft. *Ans.*

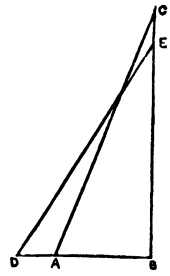
4. AC the part left standing; CAB a right angle; CB the fallen part. Given BA = 9 ft., and BC + CA = 21 ft. We have $BA^2 \div 21 = 81 \div 21 = 3\frac{6}{7} = BC - CA$; $\therefore$ $CA = \frac{1}{2}(21 - 3\frac{6}{7}) = 8\frac{4}{7}$ ft. *Ans.*

5. LMN a right angle. Given LM = 272 yds., and LN + NM = 578 yds.

We have $LM^2 \div 578 = 272 \times 272 \div 578 = 16 \times 16 \div 2 = 128 = LN - NM$;

$\therefore$ $LN = \frac{1}{2}(578 + 128) = 353$ yds. *Ans.*

6. When the side of a square is 1, the diagonal $= \sqrt{2} = 1{\cdot}4142$; $\therefore$ ·4142 of a side of the proposed square = 10 inches; hence $10 \div {\cdot}4142 = 24{\cdot}14$ inches. *Ans.*



7. AB the horizontal distance of 10 ft. from B, the foot of the wall BC; AC = the ladder, DB the horizontal distance of 14 ft. from B, and CE the distance of 2 ft. below the top of the wall; then DE = the ladder.

Now, $AC^2 - AB^2 = AC^2 - 100 = BC^2$,

and $DE^2 - DB^2 = AC^2 - 196 = BE^2$;

thus we have $BC^2 - BE^2 = 96$, and we have $BC - BE = 2$,

$\therefore$ $BC + BE = 96 \div 2 = 48$; and hence

$BC = \frac{1}{2}(48 + 2) = 25$;

$\sqrt{(AB^2 + BC^2)} = AC = \sqrt{725} = 26{\cdot}9258$ ft. *Ans.*

8. The proportionate sides 3 and 5 make the proportionate hypotenuse $= \sqrt{(9 + 25)} = \sqrt{34}$. Now the area of the triangle having this hypotenuse is $\frac{1}{2}$ of $3 \times 5 = 7\frac{1}{2}$, and this area is to the area required as the squares of the hypotenuses, that is, $34 : 35^2$; hence $7{\cdot}5 \times 1225 \div 34 = 9187{\cdot}5 \div 34 = 270{\cdot}22$ sq. in. *Ans.*

9. The sides of the triangle are as 9, 5, and $\sqrt{14 \times 4}$; the last of these proportionate quantities representing 60; hence 9 represents an hypotenuse

$$= 60 \times 9 \div \sqrt{56} = 30 \times 9 \div \sqrt{14}$$

$$\frac{30 \times 9}{\sqrt{14}} = \frac{30 \times 9 \times \sqrt{14}}{14} = \frac{\sqrt{14} \times 135}{7}$$

$$= 3{\cdot}74166 \times 135 \div 7 = 72{\cdot}1605. \quad \textit{Ans.}$$

10. $\frac{1}{2}(221 + 368 + 285) = 437$;

$\sqrt{(216 \times 69 \times 152 \times 437)}$

$= \sqrt{(72 \times 23 \times 9 \times 8 \times 19 \times 19 \times 23)}$

$= 72 \times 23 \times 19 = 31464$, the area. *1st Ans.*

$(144 \times 23 \times 19) \div 368 = 9 \times 19 = 171$. *2nd Ans.*

11. $$\frac{(385 + 281)(385 - 281)}{468} = \frac{666 \times 104}{9 \times 52} = 74 \times 2$$

$= 148$, the diff. of the segments,

468 the sum of ditto

$616 \div 2 = 308$

160 } *Ans.*

12. *Note.*—The angle A C B is found to be obtuse, and the perpendicular from B falls on A C produced.

$\frac{1}{2}(2788+2133+925)=2923$;

$\sqrt{(135\times790\times1998\times2923)}$

$=\sqrt{(9\times150\times79\times6\times9\times37^2\times79)}$

$=\sqrt{(9\times900\times79^2\times9\times37^2)}=270\times79\times37=$ the area:

$\frac{540\times79\times37}{2133}=\frac{540\times79\times37}{27\times79}=740$. *Ans.*

13. $\frac{1}{2}(171+109+100)=190$;

$\sqrt{(19\times81\times90\times190)}=19\times9\times30=$ the area.

$\frac{60\times19\times9}{171}=60=BD$

$\sqrt{(100+60)(100-60)}=80=DC$ } *Ans.*

$91=AD$

14. The sum of the perpendicular sides is to their difference as 9 to 5, $\therefore$ the less $=\frac{9-5}{9+5}$ or $\frac{2}{7}$ of the greater $=\frac{2}{7}$ of $35=10$; $\sqrt{(35^2+10^2)}=\sqrt{1325}=36{\cdot}4$. *Ans.*

15. BE perpendicular to AD is equal to AE, because each of the angles BAE and ABE is 45°; $\therefore$ $26^2=2BE^2$, or $BE^2=338$, $\therefore$ $BE=13\sqrt{2}$; then $AD=956\div13\sqrt{2}=956\sqrt{2}\div26=1{\cdot}4142\times478\div13$ $=52$ *nearly*. *Ans.*

16. BE perpendicular to AD makes the angle ABE $=30°$, the angle BAE being 60°; so that $AE=\frac{1}{2}AB$ $=7$ inches; $BE=\sqrt{(14^2-7^2)}=\sqrt{147}=7\sqrt{3}$; hence the area $=23\times7\sqrt{3}=1{\cdot}73205\times161=278{\cdot}86$ sq. in. *Ans.*

EXERCISES 3.

1. 1 lb. T$=\frac{12}{5}$ lb. C, $\therefore$ 16 lb. T$=\frac{192}{5}$ lb. C;

hence 16 lb. T + 12$\frac{3}{4}$ lb. C$=\left(\frac{192}{5}+\frac{51}{4}\right)$ lb. C

$=\frac{31}{12}$ doz. W;

$\therefore \frac{768+255}{20}$ lb. C $\times \frac{12}{31} \times 15 = 15$ doz. W

$=\frac{1023 \times 9}{31} = 33 \times 9 = 297$ lb. C. *Ans.*

2. 1 W$=\frac{4}{5}$M, $\therefore$ 1 M + 1 W$=1\frac{4}{5}$ M. The question now is, to find the time for $1\frac{4}{5}$ M corresponding to 9 hours for 16 men: this is equal to 144 hrs.$\div 1\frac{4}{5} = 80$ hrs., or $8\frac{8}{9}$ days. *Ans.*

3. 7 men and 23 boys $= 21 + 23$, or 44 boys;
23 „ 7 „ $= 69 + 7$, or 76 „
If 44 boys would take 9 hours, 76 boys would take
$\frac{44}{76}$ of 9 hrs.$= 5\frac{4}{19}$ hrs. *Ans.*

4. 10 W $= 15$ G, $\therefore$ 1 W$= 1\frac{1}{2}$ G;
12 B $= 15$ G, $\therefore$ 1 B$= 1\frac{1}{4}$ G;
hence 1 W + 1 B + 1 G$= 1\frac{1}{2} + 1\frac{1}{4} + 1 = 3\frac{3}{4}$ G.
Now, if 15 girls take 1 day, 1 girl would take 15 days;
$\therefore$ $3\frac{3}{4}$ girls would take $15 \div 3\frac{3}{4} = 4$ days. *Ans.*

5. Calling the former sort of coin F, and the latter L, we have

13 F + 8 L$= 1045d. = 9$ F + 14 L;
$\therefore$ 4 F $= 6$ L, or F$= 1\frac{1}{2}$ L;

hence $13 \times 1\frac{1}{2} + 8 = 27\frac{1}{2}$ L $= 1045d.$
or 11 L $= 418d.$, or L $= 38d. = 3s.\ 2d.$
F $= 38 \times 1\frac{1}{2} = 57d. = 4s.\ 9d.$ } *Ans.*

6. Amount for 1 man + 1 wom. $= 51s.$
do. for 13 men + 13 wom. $= 51 \times 13 = 663s.$
do. for 14 men + 13 wom. $= 696s.$
$696 - 663 = 33s.$ for a man
$51 - 33 = 18s.$ for a wom. } *Ans.*

7. 2 sofas = 4 tables, and 9 chairs = $1\frac{4}{5}$ of a table;
hence, 2 S + 3 T + 9 Ch $= 4 + 3 + 1\frac{4}{5} = 8\frac{4}{5}$ T;
again: 1 S + 1 T + 4 Ch $= 2 + 1 + \frac{4}{5} = 3\frac{4}{5}$ T;
$\frac{3 \cdot 8}{8 \cdot 8}$ or $\frac{19}{44}$ of $1276s. = 29s. \times 19 = £27\ 11s.$ *Ans.*

8. The field can be reaped in 1 day
by 14 men and 42 wom., or by 20 men and 32 wom.;
$\therefore$ 6 men = 10 wom., and 2 men = $3\frac{1}{3}$ wom.;
hence the question is: If 2 men and 6 wom., or $9\frac{1}{3}$ wom., take 7 days, what time would be taken by 6 men and 4 wom., or 14 wom.?
Here 14 wom. = $9\frac{1}{3}$ wom. $\times \frac{3}{2}$, $\therefore$ 7 da. $\div \frac{3}{2}$
$= 4\frac{2}{3}$ da. *Ans.*

9. Here $5s. \times 1st$ no. + $5s. \times 2nd$ no. $= 4\frac{3}{4}s. \times 1st$
$+ 5\frac{2}{3}s. \times 2nd.$
$\therefore (5 - 4\frac{3}{4}) \times 1st = (5\frac{2}{3} - 5) \times 2nd;$
or, $1st : 2nd :: \frac{2}{3} : \frac{1}{4}$, or as 8 : 3. *Ans.*

10.

11 times the $1st + 11$ times the $2nd = 1\frac{7}{18} \times 11 = 10\frac{7}{18}$;
8 „ „ $+ 11$ „ „ $= 7\frac{8}{9} = 7\frac{16}{18}$;
$\therefore 3$ times the $1st = 10\frac{7}{18} - 7\frac{16}{18} = 2\frac{1}{2}$; hence $1st = \frac{5}{6}$
$2nd = \frac{1}{9}$ } *Ans.*

11.

The greater $\times 19 +$ the less $\times 19 = 150 \times 19 = 2850$
„ $\times 16 -$ „ $\times 19 =$ 55
$\therefore$ 35 times the greater $=$ 2905;
hence the parts are 83 and 67. *Ans.*

12. $\frac{5}{6}$ of greater $+ \frac{5}{6}$ of less $= \frac{5}{6}$ of $1 = \cdot 83\frac{1}{3}$
$\frac{3}{4}$ „ $+ \frac{5}{6}$ „ $=$ $\cdot 77$
$\therefore$ $\frac{5}{6} - \frac{3}{4}$, or $\frac{1}{12}$ of the greater $= \cdot 06\frac{1}{3}$;
hence the parts are $\cdot 76$ and $\cdot 24$. *Ans.*

13. $1\frac{2}{3}$ wom. $= 1$ man; $1\frac{5}{6}$ boys $= 1$ wom.; hence, by reduction,

5 m. + 7 w. + 6 b.	7 m. + 6 w. + 10 b.
$1\frac{2}{3}$	$1\frac{2}{3}$
$15\frac{1}{3}$ w.	$17\frac{2}{3}$ w.
11	11
$6)168\frac{2}{3}$	$6)194\frac{1}{3}$
$28\frac{1}{9} + 6 = 34\frac{1}{9}$ b.;	$32\frac{7}{18} + 10 = 42\frac{7}{18}$ b.;

$34\frac{1}{9}$ boys would earn $\frac{1}{8}$ of $4912d. = 6140d.$ in 10 da.;
$\therefore$ $42\frac{7}{18}$ boys in 10 da. earn $\frac{6140d. \times 763}{614} = 7630d.$
$= 635s.\ 10d. = £31\ 15s.\ 10d.$ *Ans.*

14. $9\frac{9}{11}$ is to $10\frac{2}{7}$ as 21 is to 22; hence we may state as follows:—
6 men and 5 wom. do a certain amount in 21 da.;
5 men and 6 wom. do the same amount in 22 da.;
hence, in 1 day 126 m. and 105 w. do as much as
110 m. and 132 w.;
showing that 16 men are equal to 27 women. *Ans.*

15. 58 lb. and 17 doz. cost $69s.$
36 lb. $\times \frac{91\frac{2}{3}}{100}$ and 16 doz. $\times \frac{106\frac{1}{4}}{100}$ cost $50\frac{1}{4}s.$

The second of these statements simplified becomes

132 lb. + 68 doz. cost 201*s*.

(1st × 4) 232 lb. + 68 doz. cost 276*s*.

∴ 100 lb. of cod cost 75*s*., or 1 lb. of cod cost $\frac{3}{4}s$.;

and ∴ 1 doz. oysters cost $\frac{69s. - \frac{3}{4}s. \times 58}{17} = 1\frac{1}{2}s.$;

or, cod was 9*d*. a lb., and oysters 18*d*. a doz. *Ans.*

16. 12 st. B × $\frac{106\frac{2}{3}}{100}$ and 10 st. M × $\frac{95}{100}$ cost $143\frac{1}{3}s$.

5 st. B × $\frac{96}{100}$ and 6 st. M × $\frac{106\frac{1}{4}}{100}$ cost $72\frac{1}{2}s$.

These statements simplified become

384 st. B with 285 st. M cost 4300*s*.

and 384 st. B with 510 st. M cost 5800*s*.

hence 225 st. M cost 1500*s*.

or, the price of mutton was 6*s*. 8*d*. *Ans.*

∴ 285 st. mutton cost £95 = 1900*s*.

and ∴ 384 st. beef cost 4300 − 1900 = 2400*s*.

or, the price of beef was 6*s*. 3*d*. *Ans.*

EXERCISES 4.

1. Let the L.C.M. of 6 and 8, viz., 24 units be the whole work. In 1 day, A does 4 units, B 3; 7 units of work together; then 24 ÷ 7 = $3\frac{3}{7}$ days. *Ans.*

2. Assume for the whole work the L.C.M. of $5\frac{5}{6}$ and 10 = 70 units. In 1 hour A and B do 12 units, A alone does 7; ∴ B alone does 12 − 7 = 5 units per hour, or the whole in 70 ÷ 5 = 14 hrs. *Ans.*

3. If the whole work = the L.C.M. of 18, 21, 28, = 252 units, then, in 1 day A does 14 units, B 12,

C 9; 35 units together in 1 day, or the whole in $252 \div 35 = 7\frac{1}{5}$ days. *Ans.*

4. Assume as the content of the cistern the L.C.M. of 63, 72, and 80 = 5040 units of volume. In 1 minute A supplies 80 units, B discharges 70 units, and C discharges 63 units; hence $70 + 63 - 80 = 53$ units emptied per minute; hence the whole is emptied in $5040 \div 53 = 95\frac{5}{53}$ min. *Ans.*

5. Let the content of the cistern = the L.C.M. of 24, 20, and 27 = 1080 units of volume. In 1 minute A can supply 45 units, B 54, C 40. In $4\frac{1}{2}$ minutes the three supply $139 \times 4\frac{1}{2} = 625\frac{1}{2}$ units, and then $454\frac{1}{2}$ units remain to be supplied by A alone; this will be done in $454\frac{1}{2} \div 45 = 909 \div 90 = 10$ min. 6 sec. *Ans.*

6. Let the capacity of the cistern be represented by the L.C.M. of 72 and 96, viz. 288 units of volume.

The supply pipe sends in 4 units per min.,
but the cistern receives only 3 „ „
therefore the leak drains off 1 unit per min.,
or $\frac{1}{4}$ of 288 units in 216 min. = 3 hrs. 36 min. *Ans.*

7. If A's rate be 1, B's is $\frac{4}{3}$, C's $\frac{4}{5}$ of $\frac{4}{3} = \frac{16}{15}$, and D's $\frac{6}{5}$ of $\frac{16}{15} = \frac{32}{25}$; or the comparative rates of working of A, B, C, D are 75, 100, 80, 96.

Therefore B would do the work in $\frac{75}{100}$ of 60 hrs., C in $\frac{75}{80}$ of 60 hrs., D in $\frac{75}{96}$ of 60 hrs.; that is, A's time for the whole being 60 hrs., B's would be 45, C's $56\frac{1}{4}$, and D's $46\frac{7}{8}$ hrs. *1st Ans.*

Now, let the work be represented by the L.C.M. of these numbers, viz. 4500 units; then in 1 hour A does 75 units, B 100, C 80, D 96; 351 units together; hence $4500 \div 351 = 500 \div 39 = 12\frac{32}{39}$ hrs. *2nd Ans.*

8. Content of the cistern $= 12\frac{1}{4} \times \sqrt{12\frac{1}{4}} = \frac{343}{8}$ cub. ft. $(\frac{1}{10} + \frac{1}{12} - \frac{1}{15}) \times 5 = \frac{7}{12}$ of the content; and the weight of this $= \frac{1000 \text{ oz.} \times 343 \times 7}{8 \times 12} = \frac{2401000}{8 \times 12}$ $= 25010\frac{5}{12}$ oz. $= 1563$ lb. $2\frac{5}{12}$ oz. $= 13$ cwt. 107 lb. $2\frac{5}{12}$ oz. *Ans.*

9. Assume 6×5 or 30 units to represent the whole work; A and B together do $30 \div 3\frac{3}{4} = 8$ units per day, and as A's rate is equal to $\frac{6}{5}$ of B's, A does $\frac{6}{11}$ of the 8 units $= 4\frac{4}{11}$ units per day, and B the remaining $3\frac{7}{11}$ units. Hence A's time for the whole, $30 \div 4\frac{4}{11}$ $= 6\frac{7}{8}$ days, and B's, $30 \div 3\frac{7}{11} = 8\frac{1}{4}$ days. *Ans.*

10. L.C.M. of 14, 21, and $20 = 420$ units for the whole work. A does 30 units per day, B 20, C 21.

Now, $30 \times 1\frac{1}{2} + 20 \times 1 = 65$ units done before all begin to work together; therefore the rest of the work is done by the three in

$$\frac{420 - 65}{30 + 20 + 21} = \frac{355}{71}$$

$= 5$ days; $5 + 1\frac{1}{2} = 6\frac{1}{2}$ days. *Ans.*

11. The portion done by A per day is to that by B and C as 5 : 7; or A does per day $\frac{5}{12}$ of the whole daily amount.

The portion done by B per day is to that by A and C as 5 : 11; or B does per day $\frac{5}{16}$ of the whole daily amount.

Therefore C does per day $1 - \frac{5}{12} - \frac{5}{16} - \frac{13}{48}$ of the whole daily amount; and hence the rates of working are as $\frac{5}{12} : \frac{5}{16} : \frac{13}{48}$, or as 20 : 15 : 13. *Ans.*

12. Take the L.C.M. of 25, 27, and 30, viz. 1350 units, to represent the whole work. In 1 day A does 54 units, B 50, C 45; 149 units together.

Now, had B and C continued with A during the time of the actual completion, they would have augmented the 1350 units by $50 \times 6 + 45 \times 13$, making the whole amount of work done $= 2235$ units; hence $2235 \div 149 = 15$ days. *Ans.*

13.

The work is done by 6 days of 16 men = 96 da. of 1 man,
+ 8 „ 13 „ = 104 „ „
+10 „ 11 „ = 110 „ „
16)310 da. of 1 man.

1 man for 310 days = 16 men for $19\frac{3}{8}$ days. *Ans.*

14. Take the L.C.M. of 10 and 14, viz. 70 units, as the whole amount of work. A does 7 units per hour, B 5. If B had worked during the last 2 hours, the amount of work done would have been $70 + 5 \times 2 = 80$ units, done in the time in which the required 70 units were actually done, viz. in $80 \div (7 + 5) = 6\frac{2}{3}$ hrs. *Ans.*

15. Take 1584 units, the L.C.M. of 16, 18, and 22, to denote the whole work. In 1 day A does 99 units, B 88, C 72; sum, 259 units per day.

Now, if B and C had continued working with A to the end of the time in which the proposed amount of work was completed, they would have augmented that amount by $72 \times 4\frac{1}{3} + 88 \times 2 = 488$ units, making a total of 2072 units, done at the rate of 259 units per day, that is, in 8 days. *Ans.*

16. Suppose the whole work to consist of 84 units.
Units done by C : units by A and B :: 1 : 3,
$\therefore$ C does $\frac{1}{4}$ of the whole $= 21$ units.
Units done by A : units by B and C :: 5 : 7,

$\therefore$ A does $\frac{5}{12}$ of the whole $= 35$ units;
hence B does $84 - 21 - 35 = 28$ units.

The times in which A, B, C, singly, could do the whole are as $\frac{84}{35} : \frac{84}{28} : \frac{84}{21}$, or as $12 : 15 : 20$;

hence $\frac{12}{15-12}$ of $2\frac{1}{4}$ hrs. $= 9$ hrs. for A
$9 + 2\frac{1}{4} = 11\frac{1}{4}$ hrs. for B
$\frac{20}{12}$ of $9 = 15$ hrs. for C
} *Ans.*

EXERCISES 5.

1. The given distributives, reduced to the common denominator 72, are found to be as $32 : 60 : 27 : 42$; sum of the last two, 69; hence £345÷69 = £5, which multiplied by the distributives gives A £160, B £300, C £135, D £210. *Ans.*

2. Whole area = ·7854 of $71^2 = 3959{\cdot}2$, and $\frac{32}{113}$ of this is 1121·2 sq. ft. for the less segment, and therefore 2838 sq. ft. for the greater. *Ans.*

3. $20 \div 3\frac{3}{4} = 5\frac{1}{3}$ hrs. for the whole journey. Now 12 hrs. divided in the proportion of 3 : 5 shows that he had walked $4\frac{1}{2}$ hours when he looked at his watch; hence $5\frac{1}{3} - 4\frac{1}{2} = \frac{5}{6}$ of an hour longer, at $3\frac{3}{4}$ miles an hour, $= 3\frac{1}{8}$ mi. *Ans.*

4. The weights are as $1 : \frac{5}{6} : \frac{5}{8}$ of $\frac{5}{6} : 1\frac{1}{2}$ of $\frac{25}{48}$, or as $96 : 80 : 50 : 75$; sum 301; hence $86 \div 301$, or $\frac{2}{7}$, of each distributive $= 27\frac{3}{7}$, $22\frac{6}{7}$, $14\frac{2}{7}$, $21\frac{3}{7}$ cwt. *Ans.*

5. Numr. : Denr. :: 5 : 7; hence $\frac{5}{12}$ and $\frac{7}{12}$ of 129 $= \frac{53\frac{3}{4}}{75\frac{1}{4}}$. *Ans.*

6. Circumf. : Diam. :: 1 : ·31831 ; diff. ·68169 ; by which 7·5 ft. being divided gives 11·02 ft. *Ans.*

7. Base : Perp. :: 7 : 4 ; corresponding hypot. $= \sqrt{(49+16)}$;

hence $\frac{7}{\sqrt{65}}$ of $39 = \frac{21}{5}$ of $\sqrt{65} = 8{\cdot}062 \times 4\frac{1}{5}$

$= 33{\cdot}86$ in., the base, $\frac{4}{7}$ of which $= 19{\cdot}35$ in., the perp. *Ans.*

8. A's age was to B's as 5 : 2 ; diff. $= \frac{3}{5}$ of A's $= 24$; $\therefore$ A's $= 40$, B's $= 16$. *Ans.*

9. Length + breadth $= \frac{1}{2}$ of $3\frac{3}{11}$ miles $\times \frac{5\frac{3}{4}}{60}$

$= \frac{36 \text{ mi.} \times 23}{11 \times 480} = 12$ yds. $\times 23$ to be divided into parts as 15 : 8 ; hence the length $= 12$ yds. $\times 15 = 180$ yds., breadth $= 96$ yds. ; and area $= \frac{180 \times 96}{4840}$ acres $= \frac{18 \times 24}{121}$ acres $= 3$ ac. 2 r. 11 p. $7\frac{1}{4}$ yds. *Ans.*

10. If the wife's share be called 3 sums, the shares of the two sons are also 3 such sums, and those of the daughters 3 ; sum of all the shares $= £1542\frac{6}{7} \times 9$ $= £13884$; this was $\frac{97\frac{1}{2}}{100}$ or $\frac{39}{40}$ of the whole property ; hence

$$£13884 \div \tfrac{39}{40} = £14240. \quad \textit{Ans.}$$

11. A : B : C as 100 : 97 : 9700÷104, or as 2600 : 2425 : 2522 ; B—A $= 175$, and £12. 7*s.* 11*d.*÷175 $= 17d.$, which multiplied by the distributives gives for A £184. 3*s.* 4*d.*, for B £171. 15*s.* 5*d.*, for C £178. 12*s.* 10*d.* *Ans.*

12. The areas are as $\frac{1}{8} : \frac{1}{9} : \frac{1}{10}$, or as 45 : 40 : 36 ; sum 121 ; which is $\frac{1}{40}$ of 4840 sq. yds. ; hence 40

times each of the distributives = 1800, 1600, 1440 sq. yds. *Ans.*

13. If G's share be 1, W's is $\frac{5}{3}$, and H's $\frac{10}{21}$; these quantities are as 21 : 35 : 10; sum 66, which is 6 times the excess of the first above the third; therefore the whole sum is 6 times 2*s.* 9*d.* = 16*s.* 6*d.* *Ans.*

14. The three sums are to be as $\frac{1}{3\frac{1}{2}} : \frac{1}{3\frac{3}{4}} : \frac{1}{4\frac{1}{2}}$, or as $\frac{1}{7} : \frac{2}{15} : \frac{1}{9}$, or as 45 : 42 : 35; sum 122;

hence £408. 14*s.* ÷ 122, or £3. 7*s.* multiplied by each distributive gives £150. 15*s.*, £140. 14*s.*, £117. 5*s.* *Ans.*

15. Since $\frac{5}{6}$ of $\frac{2}{5}$ of the first = $\frac{5}{6}$ of $\frac{7}{8}$ of the second, and $\frac{7}{8}$ of $\frac{5}{6}$ of the second = $\frac{7}{8}$ of $\frac{4}{9}$ of the third, therefore $\frac{1}{3}$ of the first, $\frac{35}{48}$ of the second, and $\frac{7}{18}$ of the third are to be equal; and hence the parts will be as 3 : $\frac{48}{35}$: $\frac{18}{7}$, or as 35 : 16 : 30; sum 81, which is $\frac{9}{16}$ of 144 sq. in.;

hence $\frac{16}{9}$ of the distributives = $62\frac{2}{9}$, $28\frac{4}{9}$, $53\frac{1}{3}$ sq. in. *Ans.*

16. The first sum × 5 × $3\frac{1}{2}$ is to be equal to the second × 4 × 6;

or the first is to the second as $\frac{2}{35}$: $\frac{1}{48}$, or as 48 : 35; sum 83;

hence £2075 ÷ 83, or £25, multiplied by each distributive, gives £1200 and £875. *Ans.*

17. Diam. of the first × 11 = diam. of the second × 5 × $\frac{22}{7}$;

∴ the diameters are as $\frac{1}{11}$: $\frac{7}{110}$, or as 10 : 7.

Again: the diam. of the first is $\frac{55}{7} \times \frac{7}{22}$, or $\frac{35}{4}$, or $8\frac{3}{4}$ ft. longer than that of the second; hence $8\frac{3}{4}$ ft. is equal to $\frac{3}{10}$ of the diam. of

the first, and equal to $\frac{3}{7}$ of that of the second; ∴ the diameters are $8\frac{3}{4} \times \frac{10}{3}$ and $8\frac{3}{4} \times \frac{7}{3}$, or $29\frac{1}{6}$ and $20\frac{5}{12}$ feet. *Ans.*

18. The perimeter of the sector is made up of twice the radius and a fourth of the circumference = the diameter + .7854 of the diameter = 1·7854 of the diameter;

∴ the diameter = 25 ft. ÷ 1·7854 = 14 ft. *nearly*; hence the area of the sector = $\frac{1}{4}$ of $49 \times 3{\cdot}1416$
= ·7854 × 49 = 38·485 sq. ft. *Ans.*

19. The amounts of 100 for the given times are $100\frac{3}{4}$, $101\frac{9}{16}$, and 104; and these are as 1612 : 1625 : 1664, or as 124 : 125 : 128; hence the parts required are as $\frac{1}{124}$: $\frac{1}{125}$: $\frac{1}{128}$, or as 4000 : 3968 : 3875; sum 11843. Now £455·5 ÷ 11843 = £5 ÷ 130 = $£\frac{1}{26}$; hence, this multiplied by the distributives gives $£153\frac{11}{13}$, $£152\frac{8}{13}$, and $£149\frac{1}{26}$. *Ans.*

EXERCISES 6.

1. A gets 1 sum,
   B 1 such sum + 3*s.*
   C 3 such sums + 9*s.*

   147*s.* = 5 times A's + 12*s.*; ∴ 5 times A's = 135*s.*; hence A gets 27*s.*, B 30*s.*, C 90*s.* *Ans.*

2. A gets 1 sum.
   B 1 such sum − 5*s.*
   C 1 such sum − 15*s.*
   D 3 such sums − 20*s.*

   £19 = 6 times A's − £2; ∴ 6 times A's = £21; hence A gets £3 10*s.*, B £3 5*s.*, C £2 15*s.*, D £9 10*s.* *Ans.*

3. 1st part = 20 units,
2nd = 14 such units,
3rd = 34 „ − 2 ac.
$23\frac{1}{2}$ ac. = 68 units − 2 ac.;
∴ 68 units = $25\frac{1}{2}$ ac.;
hence 2 units = $\frac{3}{4}$ of an acre, and the parts are
$7\frac{1}{2}$, $5\frac{1}{4}$, $10\frac{3}{4}$ acres. *Ans.*

4. A's share = 15 units,
B's „ = 10 such units,
C's „ = 8 „
D's „ = 25 „ + 4*s.*
120*s.* = 58 units + 4*s.*; ∴ 58 units = 116*s.*;
hence 1 unit = 2*s.*; and the shares are,
A's 30*s.*, B's 20*s.*, C's 16*s.*, D's 54*s.* *Ans.*

5. Let the common result be called R:—
1st part = R − 3
2nd = R + 3
3rd = $\frac{1}{3}$R
4th = 3R
80 = $5\frac{1}{3}$ R; ∴ R = 15;
hence the parts are 12, 18, 5 and 45. *Ans.*

6. A square pole is $30\frac{1}{4}$ square yards. Call the common result R sq. yds.
1st part = R − 3
2nd = R + $2\frac{1}{4}$
3rd = $\frac{1}{6}$R
4th = 3R
$30\frac{1}{4}$ sq. yds. = $5\frac{1}{6}$ R − $\frac{3}{4}$ sq. yd.; ∴ $5\frac{1}{6}$ R = 31 sq. yds.,
hence R = 6 sq. yds., and the parts are
3, $8\frac{1}{4}$, 1, and 18 sq. yds. *Ans.*

7. $\frac{88}{100}$ of the 1st + £$10\frac{3}{5}$ = the 2nd,
and the 1st − £$10\frac{3}{5}$ = $\frac{104}{100}$ of the 2nd;

$\therefore$ $(1+\frac{88}{100})$ of the 1st = $(1+\frac{104}{100})$ of the 2nd;
hence the 1st : the 2nd as 2·04 : 1·88, or as 51 : 47,
or the 2nd is equal to $\frac{47}{51}$ of the 1st; and thus we have
$\frac{88}{100}$ of the 1st + £$10\frac{3}{5}$ = $\frac{47}{51}$ of the 1st,

or $\left(\frac{47}{51}-\frac{22}{25}\right)$ of the 1st $=\frac{53}{51\times25}$ of the 1st = £$10\frac{3}{5}$;

hence the 1st = $\frac{1}{5}$ of (51×25) = £255
and the 2nd = $\frac{47}{51}$ of the 1st = £235 } *Ans.*

8. If B's share be 100 units, C's is 107 such units, and A's 100 such units − 7*s.*;
sum, 307 units − 84*d.* = 530*d.*; $\therefore$ 307 units = 614*d.*,
and hence 1 unit being = 2*d.*, we have

A's share 200*d.* − 84*d.* = 116*d.* = 9*s.* 8*d.*
B's, 200*d.* = 16*s.* 8*d.* } *Ans.*
C's, 214*d.* = 17*s.* 10*d.*

9. Write D to signify the difference of the terms:—
11 − 5 : 5 :: D : $\frac{5}{6}$D = the numerator + 1;
7 − 3 : 3 :: D : $\frac{3}{4}$D = the numerator − 1;

hence $\frac{1}{12}$D is equal to 2, and D = 24;
11 − 5 : 5 :: 24 : 20 = the numerator + 1;
hence the numerator is 19, and the fraction is $\frac{19}{43}$. *Ans.*

EXERCISES 7.

1. B gains on A 2 miles an hour, and therefore will gain 8 miles in 4 hours; he has then 16 miles to gain, for which he goes 3 miles an hour faster than A; he will now, therefore, overtake A in 16 ÷ 3, or $5\frac{1}{3}$ hours. *Ans.*

2. $(40+35)$ miles an hour is $\frac{1760 \text{ yds.} \times 75}{3600}$ or $36\frac{2}{3}$ yards per second; so the combined lengths of the trains are $=36\frac{2}{3} \times 4{\cdot}8 = 11 \times 16 = 176$ yards; hence $176-90=86$ yds. *1st Ans.*

Again: the faster train would gain 5 miles, or 8800 yards, per hour, or $2\frac{4}{9}$ yards per second, on the slower train, and would clear the 176 yards in $176 \div 2\frac{4}{9} = 72$ seconds. *2nd Ans.*

3. A having done $2\frac{1}{2}$ yds. $\times 1\frac{1}{2}$, or $3\frac{3}{4}$ yards, before B begins, the remaining $318\frac{1}{4}$ yds. are done between them in $318\frac{1}{4} \div (2\frac{1}{2}+2\frac{7}{9}) = \frac{1273}{4} \times \frac{18}{95} = \frac{67 \times 9}{2 \times 5} = 60\frac{3}{10}$ hrs., to which add A's $1\frac{1}{2}$ hour, making the whole time 61 hrs. 48 min. *Ans.*

4. Call the hour and minute hands H and M. At 10 o'clock M is 10 minute spaces in advance of H; therefore M has to gain 5 minute spaces farther beyond H, that the two hands may be at right angles to each other. Now as in 60 minutes of time M passes over 60 spaces, and H over 5, M gains on H 55 spaces per hour, and will therefore gain 5 spaces in $\frac{5}{55}$ or $\frac{1}{11}$ of an hour, which will be at $5\frac{5}{11}$ minutes past 10.

But there will be another occurrence, between 10 and 11 o'clock, of an interval of 15 minute spaces between the hands. At 10 o'clock H is 50 minute spaces in advance of M, and will be only 15 spaces in advance when M has gained $50-15$ spaces, which will be in $\frac{35}{55}$ or $\frac{7}{11}$ of an hour, that is, at $38\frac{2}{11}$ min. past 10.

The hands, therefore, will be at right angles at 10 hrs. $5\frac{5}{11}$ min., and again at 10 hrs. $38\frac{2}{11}$ min. *1st Ans.*

Secondly. The hands will be coincident, when M has gained 50 spaces on H, viz. in $\frac{50}{55}$ or $\frac{10}{11}$ of an hour, viz. at $54\frac{6}{11}$ minutes past 10. *2nd Ans.*

Thirdly. They will point in opposite directions, that is, the interval between them will be equal to 30 minute spaces, when M has gained 50 − 30 spaces, which will be in $\frac{20}{55}$ or $\frac{4}{11}$ of an hour, viz. at $21\frac{9}{11}$ min. past 10. *3rd Ans.*

5. M gains on L $10\frac{3}{4} - 8\frac{1}{4}$ or $2\frac{1}{2}$ miles an hour, and has to gain on him in all $\frac{2}{6}\frac{5}{0}$ or $\frac{5}{12}$ of $8\frac{1}{4}$ miles, which he will accordingly do in $8\frac{1}{4} \times \frac{5}{12} \div 2\frac{1}{2} = \frac{1}{6}$ of $8\frac{1}{4}$ hours $= 1\frac{3}{8}$ hours, in which time each will have travelled $10\frac{3}{4} \times 1\frac{3}{8} = 14\frac{2}{3}\frac{5}{2}$ miles. *Ans.*

6. A's rate is 2 units of distance per hour, B's 3 such units; hence $5 \times 4 = 20$ units for the whole distance. Therefore A's time for the whole journey is $20 \div 2 = 10$ hrs., and B's $20 \div 3 = 6\frac{2}{3}$ hrs. *Ans.*

7. B and C meet in $25\frac{1}{3} \div (3\frac{3}{4} + 4\frac{1}{6}) = 3\frac{1}{5}$ hours; hence B, when he overtook A, had gone $3\frac{3}{4} \times 3\frac{1}{5} = 12$ miles; which distance was gone by A in $12 \div 3\frac{1}{3}$ or $3\frac{3}{5}$ hours;

$\therefore$ B started $3\frac{3}{5} - 3\frac{1}{5}$ or $\frac{2}{5}$ of an hour, that is, 24 minutes later than A. *Ans.*

8. In order to meet, we have

10 mi. × A's no. of hrs. riding = $11\frac{1}{9}$ × B's no. of hours;

or, A's no. of hours : B's as $11\frac{1}{9}$: 10, or as 10 : 9;

hence $13\frac{1}{5}$ minutes $= \frac{1}{10}$ of A's time; or A's time of

riding to meet B = 138 minutes = $2\frac{3}{10}$ hours, which at 10 miles an hour = 23 miles. *Ans.*

9. 1*st.* They will be all again at the starting-point in the L.C.M. of $9\frac{1}{2}$, $5\frac{7}{10}$, and $4\frac{1}{2}$ minutes, equal to half the L.C.M. of 19, 57, and $9 = 85\frac{1}{2}$ minutes. 1*st Ans.*

2*nd.* In 1 minute A goes $\frac{2}{19}$ of a round, B $\frac{10}{57}$, C $\frac{2}{9}$; hence in 1 minute B gains on A $\frac{4}{57}$ of a round, or a whole round in $14\frac{1}{4}$ minutes, and C gains on A $\frac{20}{171}$ of a round, or a whole round in $8\frac{11}{20}$ minutes. Therefore A, B, and C will first be together in the L.C.M. of $14\frac{1}{4}$ and $8\frac{11}{20}$ minutes, equal to $\frac{1}{4}$ of the L.C.M. of 57 and $171 = 42\frac{3}{4}$ min. 2*nd Ans.*

10. When A starts, he has to run 9755 feet, and B 9650 feet to regain the starting point.

Now A runs 4850 feet in the time B runs 4800 feet; so that A's rate is equal to $\frac{97}{96}$ of B's. Therefore, when B will have run 9650 feet and regained the starting-point, A will have run $1\frac{1}{96}$ of 9650 feet $= 9750\frac{25}{48}$ feet, and have yet to run $9755 - 9750\frac{25}{48}$ or $4\frac{23}{48}$ feet to reach it. Thus B comes first, and is 4 ft. $5\frac{3}{4}$ in. from A. *Ans.*

11. Length of the walk, 17600 yards. When A has walked half of it, viz. 8800 yards, B has walked 8800 − 880 or 7920 yards; therefore, at first, B's rate is equal to $\frac{9}{10}$ of A's.

Afterwards, A being 8800 yards from the goal and B 9680, A's rate is to be equal to $\frac{9}{10}$ of B's. Hence, when B has walked 9680 yards, and reached the goal, A has gone $\frac{9}{10}$ of $9680 = 8712$ yards, which is $8800 - 8712 = 88$ yards short of the goal. Thus B wins by 88 yards. *Ans.*

12. A's rate : C's as 4 : 5, therefore A's time of going the whole distance is equal to $\frac{5}{4}$, or $1\frac{1}{4}$ of C's; but A's whole time is longer than C's by $45+27=72$ minutes $=1\frac{1}{5}$ hour;

$\therefore$ C's whole time $=4\frac{4}{5}$ hours; and hence B's time, 27 minutes longer, is $4\frac{4}{5}+\frac{9}{20}=5\frac{1}{4}$ hours, which at 4 miles an hour makes the whole distance 21 miles. *Ans.*

13. A rode from P to Q in $2\frac{2}{3}+2\frac{14}{15}=5\frac{3}{5}$ hours. When they met A had gone $\frac{2\frac{2}{3}}{5\frac{3}{5}}$ or $\frac{10}{21}$ of his whole journey, and therefore B had then gone $\frac{11}{21}$ of that distance $=8\frac{1}{4}$ mi. $\times 2\frac{2}{3}=22$ miles.

A's rate : B's as 10 : 11; hence $\frac{10}{11}$ of $8\frac{1}{4}=7\frac{1}{2}$ miles an hour by A. *1st Ans*; and the distance from P to Q $=\frac{21}{11}$ of $22=42$ miles. *2nd Ans.*

EXERCISES 8.

1. He sells for 28*s.* what cost him 25*s.*, and therefore gains 3 on 25, or 12 per cent. *Ans.*

2. He buys a lemon at $\frac{13}{16}$ of a penny, and sells it at $\frac{17}{12}$ of a penny; hence, prime cost is to selling price as 39 : 68; his gain, therefore, is $\frac{29}{39}$ of 36 times $\frac{13}{16}$ of a penny $=\frac{29}{16}$ of a shilling, or 1*s.* $9\frac{3}{4}$*d.* *Ans.*

3. £14. per cwt. is 2*s.* 6*d.* per lb., and a profit of $3\frac{3}{4}$*d.* on 2*s.* 6*d.* is $12\frac{1}{2}$ per cent.; hence the quantity if

sold at $7\frac{1}{2}$ per cent. profit, or at $\frac{107\frac{1}{2}}{100}$ of 30*d.* per lb., would have brought $13383\frac{3}{4}d.$

$\frac{5\,3\,5\,3\,5}{1\,2\,0} \times \frac{4\,0}{4\,3} = \frac{1\,7\,8\,4\,5}{4\,3} = 415$ lb., or 3 cwt. 79 lb. *Ans.*

4.
$$\begin{aligned} 3150 \text{ at } 1{\cdot}05 \text{ of prime cost} &= 3307{\cdot}5 \\ 1050 \text{ at } 1{\cdot}06 &= 1113 \\ 630 \text{ at } 1{\cdot}08 &= 680{\cdot}4 \\ 450 \text{ at } 1{\cdot}12 &= 504 \\ 1020 \text{ at } {\cdot}97 &= 989{\cdot}4 \\ & \quad 3286{\cdot}8 \end{aligned}$$

$241s.\ 6d. \div 20{\cdot}7$

$= 2415s. \div 207 = 11s.\ 8.$ *Ans.*

5. Profit per gallon $= \frac{2}{7}$ of (prime cost + profit);
hence $\frac{5}{7}$ of the profit $= \frac{2}{7}$ of the prime cost,
or, the profit $= \frac{2}{5}$ of the prime cost;
$\therefore$ $19s.\ 2d. \times 1\frac{2}{5} = 26s.\ 10d.$ *Ans.*

6. First, 100 becomes 200, then 200 becomes 160, then 160 becomes 128, and lastly, 128 becomes $102\frac{2}{5}$, showing a profit of $2\frac{2}{5}$ on the original 100.

7. Prime cost $\times 8\frac{1}{4}$ = selling price $\times 6\frac{2}{7}$;
$\therefore$ prime cost : selling price as $6\frac{2}{7} : 8\frac{1}{4}$, or as 16 : 21;
and therefore, $\frac{5}{16}$ of $100 = 31\frac{1}{4}$ per cent. *Ans.*

8. 20 : 23 as 100 : 115;
and 115 : 15 as 100 : $13\frac{1}{23}$ per cent. *Ans.*

9. 10 sheep at 10*s.* would amount to an outlay of 100*s.*;
9 sheep sold at 12*s.* would produce 108*s.*;
hence, if 8*s.* be gained on 100*s.*, then 72*s.* would be the gain on 900*s.*; and $\sqrt{900} = 30$. *Ans.*

10. If the sum paid for the goods by A be called 1,
then 1·025 = sum paid by B,
1½ p. c. = ·015375
1·009625 = sum paid by C,
1¼ p. c. = ·012620
1·022245 = sum paid by D,
2 p. c. = ·020445
1·001800, received by D,
= ·0018 more than A's proportionate unit;
∴ 18*s.* ÷ ·0018 = 10000*s.* = £500. *Ans.*

11. 29*s.* 6*d.* a hundred is $\frac{2}{80}$*s.* for each cigar; hence 30 guineas is the cost of $630 \div \frac{2}{80}$, or 2400 cigars, or 200 dozen; ∴ he must sell 200 doz. − 25 doz. = 175 doz. for 630*s*, which is at the rate of 5 dozen for 18*s.* *Ans.*

12. $\frac{92\frac{1}{2}}{100}$ of the invoiced or professed selling price is to be equal to $\frac{112\frac{1}{2}}{100}$ of 74*s.*; hence the selling price is

$$= \frac{112\frac{1}{2}}{92\frac{1}{2}} \text{ of } \frac{45}{34} \text{ of } 74s. = 90s. \quad Ans.$$

13. Present value of 3*s.* for ¼ of a year at 5 per cent.

$$= \frac{100}{101\frac{1}{4}} \text{ or } \frac{80}{81} \text{ of } 3s. = \frac{80}{27} \text{ of a shilling; hence}$$

$$2\tfrac{2}{3}s. : \frac{80}{27}s. \text{ as } 100 : \frac{100 \times 80 \times 3}{27 \times 8} = 111\tfrac{1}{9};$$

showing the gain at the moment of sale to be 11⅑ per cent. *Ans.*

14. Present value of 30*s.* for ¼ of a year at 5 per cent.

$= \frac{100}{101\frac{1}{4}}$ or $\frac{80}{81}$ of $30s. = \frac{800}{27}$ of a shilling.

Proposed selling price $= \frac{107\frac{1}{2}}{100}$ of $\frac{800}{27}s.$, the amount of which in $\frac{3}{4}$ of a year is $\frac{103\frac{3}{4}}{100}$ of $\frac{107\frac{1}{2}}{100}$ of $\frac{800}{27}s.$,

$= \frac{83}{80}$ of $\frac{43}{40}$ of $\frac{800}{27}s. = 33s.\ 0\frac{5}{9}d.$ *Ans.*

15. Prime cost $= \frac{100}{97\frac{1}{2}}$ or $\frac{40}{39}$ of $9\frac{3}{4}s. = 10s.$

$\frac{109\frac{1}{11}}{100}$ of $10s. = \frac{120}{11}s.$, the present value of $11s.$ for the time sought;

$\frac{121}{120}$ of $100 = \frac{605}{6} = 100\frac{5}{6}$; this shows that $£\frac{5}{6}$ is the whole interest on 100 at 5 per cent.; and hence the term of credit $= \frac{1}{6}$ of a year or 2 months. *Ans.*

EXERCISES 9.

1. £8. 8*s*. 3*d*. is $\frac{1}{4}$ per cent. on the amount of stock;
∴ 400 times $168\frac{1}{4}s. = 20$ times $£168\frac{1}{4} = £3365$.
Ans.

2. $£3\frac{1}{2} \times \frac{30 \text{ cents.} \times 89\frac{1}{4}}{102} = \frac{7 \times 15 \times 357}{102 \times 4} = \frac{35 \times 21}{8} = £91\frac{7}{8}$;

thus the second income is £91 17*s*. 6*d*.,
first income £3 × 30 cents. = £90 0*s*. 0*d*.
An increase of £1 17*s*. 6*d*. *Ans.*

3. He buys $\frac{9065}{97\frac{1}{8}}$ cents., which he sells at £100 cash for each cent., and with this amount of cash he buys

Consols at £94½ cash for each cent. of that stock; hence he obtains

$$\frac{9065\times100}{97\frac{1}{8}\times94\frac{1}{2}}\text{ cents.}=\frac{72520\times200}{777\times189}=\frac{10360\times200}{111\times189}$$

$$=\frac{1480\times200}{37\times81}=\frac{8000}{81}\text{ cents.}=£9876\tfrac{44}{81}.\quad \textit{Ans.}$$

4. $\frac{3}{5}$ of the 1st income $=\frac{3}{5}$ of £4 × 20½ cents.
$\frac{3}{5}$ of the 2nd income $=\frac{3}{5}$ of £3 × 20½ cents.

$$\times\frac{112\frac{1}{2}}{82}.$$

$$\text{Difference}=\left(\frac{£337\frac{1}{2}}{82}-£4\right)\times\frac{41\times3}{10}$$

$$=\tfrac{3}{20}\text{ of }(£337\tfrac{1}{2}-£328)$$

$$=9\tfrac{1}{2}s.\times3=28s.\ 6d.\quad \textit{Ans.}$$

5. $\frac{100}{120}$ of £6 = 5 per cent. *Ans.*

6. The sum $\times\left(\frac{4\frac{1}{2}}{101\frac{1}{4}}-\frac{4}{100}\right)=£16\frac{2}{5}$;

$$\therefore\ £\frac{82}{5}\div\left(\frac{2}{45}-\frac{1}{25}\right)=\frac{£82\times45\times25}{5\times5}=£3690.$$

Ans.

7. £750 ÷ £3 = 250 cents.

$$\text{The rate sought}\times\frac{125\times96}{120}=£375+£50;$$

$$\therefore\ \text{the rate}=\frac{425\times120}{1000\times12}=4{\cdot}25,\text{ or }4\tfrac{1}{4}\text{ per cent.}$$

Ans.

8. Difference of incomes per £1 of investment $=\frac{5}{117\frac{1}{2}}-\frac{3}{92\frac{1}{4}}=\frac{2}{47}-\frac{4}{123}=\frac{58}{47\times123}$ of £1; and his money multiplied by that difference = £29; hence his money is equal to $\frac{29}{58}$ of £47 × 123 = £2890. 10*s.* *Ans.*

9. Gross income $= \frac{240}{235}$ of £$452\frac{3}{8} = \frac{£3619 \times 48}{8 \times 47}$
$= £77 \times 6$; and this divided by $3£ = 154$ cents.

2nd income $= £4 \times \frac{154 \times 78\frac{1}{2}}{102\frac{1}{2}} = \frac{£616 \times 157}{205}$
$= £471.\ 15s.\ 4d.$ *nearly*;
hence £471. 15*s.* 4*d.* − £462 = £9. 15*s.* 4*d.*, better.
Ans.

10. The sum he has to invest is

$$£187\tfrac{1}{2} \div \left(\frac{100}{84} - \frac{100}{96}\right),$$

and the difference of incomes is $= \left(\frac{3\frac{1}{2}}{96} - \frac{3}{84}\right)$ of that sum,

$$\frac{6}{96 \times 84} \text{ of } \frac{£375}{2} \times \frac{84 \times 96}{1200} = \frac{£375}{400} = \frac{375s.}{20}$$
$$= 18s.\ 9d. \quad \textit{Ans.}$$

11. $\frac{80\frac{3}{4}}{100}$ of $5\frac{5}{19} = 80\frac{3}{4} \div 19 = 4\frac{1}{4}$ per cent. stock.
Ans.

12. 1st income $= \frac{18550}{92\frac{3}{4}}$ cents. at £$3\frac{1}{2} = \frac{2}{5\,3}$ of £18550
$= £700$;

2nd income $= \frac{2}{5}$ of $\frac{18550 \times 4}{96} + \frac{3}{5}$ of $\frac{18550 \times 3}{90}$,
$= (\frac{1}{60} + \frac{1}{50})$ of $18550 = £680.\ 3s.\ 4d.$,
which is £19. 16*s.* 8*d.* less than before. *Ans.*

13. $\frac{3}{200}$ of the original stock is invested at $93\frac{3}{4}$, and brings additional stock $= \frac{100}{93\frac{3}{4}}$ of $\frac{3}{200}$ or $\frac{2}{125}$ of the original stock; hence the next half-year's dividend $= \frac{3}{200}$ of $1\frac{2}{125}$ of the original stock = £381;

so that the original stock is

$$= £381 \div \frac{381}{25000} = £25000. \quad Ans.$$

14. No. of cents. bought $= 2345 \div 87\frac{1}{2} = 26\frac{4}{5}$;
2nd income $= £3\frac{1}{4} \times 26\frac{4}{5} - £\frac{7}{10} = £86\frac{2}{5}$;
$26\frac{4}{5}$ cents. sold out at 90 produces £2412 cash.
No. of cents. of 3 per cent. stock bought $= 86\frac{2}{5} \div 3$
$= 28\frac{4}{5}$;
hence the price $= \frac{2412}{28{\cdot}8} = \frac{1005}{12} = 83\frac{3}{4}$. *Ans.*

15. The 1st income $= £3 \times 102\frac{3}{4\,0}$ cents. $= £307.\ 14s.\ 6d.$
The anticipated income was $£318\frac{7}{4\,0}$;
$\therefore\ 318\frac{7}{4\,0} \div 3\frac{1}{4} = 97\frac{9}{1\,0}$ cents. at $93\frac{1}{4}$ was the cash value of the $102\frac{3}{4\,0}$ cents. according to which the increase of income was calculated, and the price of stock was then $= 97{\cdot}9 \times 93\frac{1}{4} \div 102\frac{3}{4\,0} = 979 \times 373 \div 4103$
$= 979 \div 11 = 89$; hence he sold out at $89\frac{1}{4}$ and reinvested at $93\frac{1}{2}$, and his 2nd income was

$$= \frac{102\frac{3}{4\,0} \times 89\frac{1}{4} \times 3\frac{1}{4}}{93\frac{1}{2}} = \frac{4103 \times 357 \times 13}{187 \times 320} = \frac{373 \times 21 \times 13}{320}$$

$= £318.\ 4s.\ 3\frac{3}{4}d.$, being an increase of $£10.\ 9s.\ 9\frac{3}{4}d.$
Ans.

16. If only £5170 had been reinvested the 2nd income would have been $= £5170 \times \frac{3\frac{1}{4}}{77} = £235 \times \frac{13}{14}$; and if we suppose the first income for each cent., viz. £4, to have been reduced by $\frac{3\frac{1}{4}}{77}$ of the income for each cent. in $£3\frac{1}{8}$, there would have been $£5\frac{1}{3}\frac{5}{2}$ less income than the above; hence the no. of cents. bought at first

$$= \frac{£235 \times \frac{13}{14} - £5\frac{15}{32}}{£4 - £3\frac{1}{8} \times \frac{3\frac{1}{4}}{77}} = \frac{3760 \times 1\frac{6}{7} - 175}{128 - 25 \times \frac{13}{77}}$$

$$= \frac{11\,(3760 \times 13 - 1225}{128 \times 77 - 25 \times 13} = \frac{55\,(9776 - 245)}{9856 - 325} = 55 \text{ cents.};$$

and hence the price of stock was $5170 \div 55 = 94$. *Ans.*

EXERCISES 10.

1. Area of walls $(27 + 17) \times 2 \times 16\frac{1}{4} = 22 \times 65$
$= 1430$ sq. ft.
$= \frac{1}{9}$ of 1430 sq. yds.; hence the length of paper
$= \frac{1430}{9 \times \frac{5}{8}}$ yds. at $4\frac{1}{2}d.$, $\frac{4\frac{1}{2}d. \times 1430 \times 8}{9 \times 5} = 143d. \times 8$
$= 1144d. = 95s.\ 4d. = £4.\ 15s.\ 4d.$ *Ans.*

2. Area of walls $(21\frac{1}{3} + 14\frac{2}{3}) \times 2 \times 14 = 36 \times 28$ sq. ft.
$= 4 \times 28$ sq. yds.; hence $112 - 17$, or 95 sq. yds. of paper divided by $\frac{19}{24}$ of a yard wide $= 120$ yds., length of paper. *Ans.*

3. Area to be papered $= \frac{5}{6}$ of 39×23 sq. ft., which divided by $1\frac{1}{4}$ ft. width $= 26 \times 23$ ft. length of paper at $2\frac{1}{2}d.$ a foot $= 65d. \times 23 = 1495d. = 124s.\ 7d. = £6.\ 4s.\ 7d.$
Ans.

4. Area of walls $= 36\frac{3}{4} \times 25$ sq. ft.; $\therefore$ the length of paper $= 36\frac{3}{4} \times 25 \div 1\frac{1}{6}$ feet $= \frac{6}{7}$ of $12\frac{1}{4} \times 25$ yards $= 10\frac{1}{2} \times 25$ yds.; hence cost per yard $= \frac{1050d.}{10\frac{1}{2} \times 25}$
$= \frac{10500d.}{105 \times 25} = 4d.$ *Ans.*

5. Area of walls, 48×25 sq. ft.; length of paper, $1760d. \div 5\frac{1}{2}d. = 320$ yds., or 960 ft.; hence width of paper, $48 \times 25 \div 960 = 1\frac{1}{4}$ ft., or 15 inches. *Ans.*

6. Length + width = 15 yds., and the length is to the width as $1\frac{1}{2}$: 1 or as 3 : 2; hence the length and width are $\frac{3}{5}$ and $\frac{2}{5}$ of 15, or 9 yds. and 6 yds.

Length of paper = 1566*d.* ÷ 3*d.* = 522 yds., therefore the square yards of paper = $522 \times \frac{10}{36} = 145$; hence the height = $145 \div 30 = 4\frac{5}{6}$ yards, or $14\frac{1}{2}$ ft.

Thus the dimensions are 27 feet long, 18 wide, $14\frac{1}{2}$ high. *Ans.*

7. Area of ceiling or floor = $453\frac{3}{4}d. \div 7\frac{1}{2}d. = 1815 \div 30 = 60\frac{1}{2}$ sq. yds. = length × width = twice the square of the width; ∴ the square of the width is $30\frac{1}{4}$ sq. yds., and hence the width is $5\frac{1}{2}$ yds., and the length 11 yds.

Area of walls = $586\frac{2}{3}s. \div 3\frac{1}{3}s. = 1760 \div 10 = 176$ sq. yds.; and this is equal to $(11 + 5\frac{1}{2}) \times 2 \times$ the height in yards; ∴ the height = $176 \div 33 = 5\frac{1}{3}$ yds., or 16 feet. *Ans.*

8. Entire circuit of walls = (30 + 26) ft. × 2 = 112 ft.; 84*s.* ÷ 3*s.* 6*d.* = 24 pieces = 288 yds., or 864 ft. of paper; hence the whole area of the walls = $864 \times 1\frac{3}{4} = 216$ sq. ft. × 7 = 112 ft. × the height in feet;

∴ the height = $216 \div 16 = 13\frac{1}{2}$ feet. *Ans.*

9. 87*s.* 6*d.* ÷ $4\frac{1}{2}d.$ a yard = $233\frac{1}{3}$ yds., or 700 ft., length of paper; area of walls = $700 \times 1\frac{1}{4} = 875$ sq. ft.

(8 + 6) × 2 × 5 = 140 square units = 875 sq. ft.

Hence 1 square unit = $6\frac{1}{4}$ sq. ft., the root of which, viz., $2\frac{1}{2}$ ft. is a lineal unit; and multiplying this by the proportionate length and width, we find the room to be 20 ft. long and 15 ft. broad.

Hence the length of carpet required = $20 \times 15 \div 2\frac{1}{4} = 133\frac{1}{3}$ ft., which, at 3*s.* 9*d.* a yard, or 15*d.* a foot, comes to 2000*d.* = 166*s.* 8*d.* = £8. 6*s.* 8*d.* *Ans.*

EXERCISES 11.

1.

$28\frac{1}{12}$ deg. is between 20° of 65 mi. and 30° of 60 mi.

$$60 \text{ mi.} + \frac{30 \text{ deg.} - 28{\cdot}583 \text{ deg.}}{10 \text{ deg.}} \text{ of } 5 \text{ mi.} = 60{\cdot}7, \text{ or } 61 \text{ mi.}$$

Ans.

2.

$23\frac{2}{15}$ deg. is between 20° of 65 mi. and 30° of 60 mi.

$$60 \text{ mi.} + \frac{30 \text{ deg.} - 23{\cdot}133 \text{ deg.}}{10 \text{ deg.}} \text{ of } 5 \text{ mi.} = 63{\cdot}4 \text{ mi.}$$ *Ans.*

3.

65 deg. is between 60° of $34\frac{1}{2}$ mi. and 70° of $23\frac{1}{2}$ mi.

$$23\frac{1}{2} \text{ mi.} + \frac{70° - 65°}{10°} \text{ of } 11 \text{ mi.} = 29 \text{ miles.}$$ *Ans.*

4.

56 deg. is between 50° of 44 mi. and 60° of $34\frac{1}{2}$ mi.

$$34\frac{1}{2} \text{ mi.} + \frac{60° - 56°}{10°} \text{ of } 9\frac{1}{2} \text{ mi} = 38 \text{ mi.}$$ *Ans.*

5.

41 deg. is between 40° of 53 mi. and 50° of 44 mi.

$$44 \text{ mi.} + \frac{50° - 41°}{10°} \text{ of } 9 \text{ mi.} = 52 \text{ mi.}$$ *Ans.*

6.

$12\frac{3}{4}$ deg. is between 10° of 68 mi. and 20° of 65 mi.

$$65 \text{ mi.} + \frac{20 \text{ deg.} - 12\frac{3}{4} \text{ deg.}}{10 \text{ deg.}} \text{ of } 3 \text{ mi.} = 67 \text{ mi.}$$ *Ans.*

7.

$41\frac{13}{15}$ deg. is between 40° of 53 mi. and 50° of 44 mi.

$$44 \text{ mi.} + \frac{50 \text{ deg.} - 41{\cdot}86 \text{ deg.}}{10 \text{ deg.}} \text{ of } 9 \text{ mi.} = 51 \text{ mi.}$$ *Ans.*

8.

$48\frac{5}{6}$ deg. is between 40° of 53 mi. and 50° of 44 mi.

44 mi. + $\frac{50 \text{ deg.} - 48{\cdot}83 \text{ deg.}}{10 \text{ deg.}}$ of 9 mi. = 45 mi. *Ans.*

9.

78 deg. is between 70° of $23\frac{1}{2}$ mi. and 80° of 12 mi.

12 mi. + $\frac{80 \text{ deg.} - 78 \text{ deg.}}{10 \text{ deg.}}$ of $11\frac{1}{2}$ mi. = 14 mi. *Ans.*

10.

$19\frac{2}{15}$ deg. is between 10° of 68 mi. and 20° of 65 mi.

65 mi. + $\frac{20 \text{ deg.} - 19{\cdot}133 \text{ deg.}}{10 \text{ deg.}}$ of 3 mi. = $65\frac{1}{4}$ mi. *Ans.*

11.

$63\frac{1}{2}$ mi. is between 60 mi. for lat. 30° and 65 mi. for lat. 20°.

20° + $\frac{65 \text{ mi.} - 63\frac{1}{2} \text{ mi.}}{5 \text{ mi.}}$ of 10° = 23° S. lat. *1st Ans.*

63·5 × 15° = 952 miles an hour. *2nd Ans.*

12.

$56\frac{3}{4}$ mi. is between 53 mi. for 40° and 60 mi. for 30°.

30° + $\frac{60 \text{ mi.} - 56\frac{3}{4} \text{ mi.}}{7 \text{ mi.}}$ of 10° = 34° S. *1st Ans.*

56·75 × 15° = 851 miles an hour. *2nd Ans.*

13.

46 mi. is between 44 mi. for 50° and 53 mi. for 40°.

40° + $\frac{53 \text{ mi.} - 46 \text{ mi.}}{9 \text{ mi.}}$ of 10° = 48° N. *1st Ans.*

46 mi. × 15° = 690 miles an hour. *2nd Ans.*

14.

59 mi. is between 53 mi. for 40° and 60 mi. for 30°.

30° + $\frac{60 \text{ mi.} - 59 \text{ mi.}}{7 \text{ mi.}}$ of 10° = 31° N. *1st Ans.*

-- mi. × 15° = 885 miles an hour. *2nd Ans.*

15.

$54\frac{1}{2}$ mi. is between 53 mi. for 40° and 60 mi. for 30°.

$30° + \frac{60 \text{ mi.} - 54\frac{1}{2} \text{ mi.}}{7 \text{ mi.}}$ of $10° = 38°$ S. *1st Ans.*

54·5 mi. $\times$ 15° = 817 miles an hour, *2nd Ans.*

16.

35·8 mi. is between $34\frac{1}{2}$ mi. for 60° and 44 mi. for 50°.

$50° + \frac{44 \text{ mi.} - 35{\cdot}8 \text{ mi.}}{9\frac{1}{2} \text{ mi.}}$ of $10° = 58°$ N. *1st Ans.*

35·8 mi. $\times$ 15° = 537 miles an hour. *2nd Ans.*

17.

Length of a degree in the lat. of London = 43 mi. (*see* § 5).

$43 \times 1\frac{1}{2} = 64\frac{1}{2}$ mi. the length of a degree in the lat. of Surat.

$64\frac{1}{2}$ mi. is between 60 mi. for 30° and 65 mi. for 20°.

$20° + \frac{65 \text{ mi.} - 64\frac{1}{2} \text{ mi.}}{5 \text{ mi.}}$ of $10° = 21°$ N. *Ans.*

18.

$49\frac{2}{3}$ deg. is between 40° of 53 mi. and 50° of 44 mi.

$44 \text{ mi.} + \frac{50 \text{ deg.} - 49\frac{2}{3} \text{ deg.}}{10 \text{ deg.}}$ of 9 mi. = 44·3 mi.

$44{\cdot}3 \times 1\frac{1}{3} = 59$ mi. in the latitudes sought.

59 mi. is between 53 mi. for 40° and 60 mi. for 30°.

$30° + \frac{60 \text{ mi.} - 59 \text{ mi.}}{7 \text{ mi.}}$ of $10° = 31°$ N. and S. *Ans.*

EXERCISES 12.

1. S. is 17° 33′ north of M.; hence 69 mi. $\times$ 17·55 = 1210 mi. *Ans.*

2. Diff. of long., $5 + 14\frac{1}{2} = 19\frac{1}{2}$ deg.
   Length of a degree on the parallel of 36°.

$$53 \text{ mi.} + \frac{40-36}{10} \text{ of } 7 \text{ mi.} = 55{\cdot}8 \text{ mi.}$$

$55{\cdot}8$ mi. $\times 19\frac{1}{2} = 1{,}088$ mi. *Ans.*

3. 69 mi. $\times (49-41) = 550$ mi. *Ans.*

4. Diff. of long. $1\frac{1}{15}$ deg. in lat. $40\frac{5}{6}$ deg. where the length of a degree $= 44 \text{ mi.} + \frac{50-40\frac{5}{6}}{10}$ of $(53-44)$ mi.
$= 52{\cdot}2$ mi.

In figure, p. 77, let E be the position of Rome, F that of Naples.

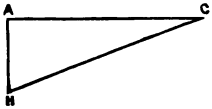
$\frac{52{\cdot}2}{69} \times \frac{26}{15} = 1{\cdot}31$ max. degrees $=$ DF; and we have DE $= 1\frac{1}{30}$ max. degrees; hence
$\sqrt{(DE^2+DF^2)} = \sqrt{2{\cdot}7864} = 1{\cdot}67$ deg. of 69 miles for EF $= 115$ miles. *Ans.*

5. In figure, p. 76, let D be the position of Lisbon, E that of Barcelona; DF the diff. of long. $= 11\frac{1}{6}$ deg. on a parallel of 38° 42′, where a degree of long.

$$= 53 \text{ mi.} + \frac{40-38{\cdot}7}{10} \text{ of } 7 \text{ mi.} = 53{\cdot}9 \text{ mi.};$$

hence $DF = \frac{53{\cdot}9}{69} \times 11\frac{1}{6} = 8{\cdot}723$ max. deg.; and we have FE, the diff. of lat., $= 2{\cdot}3$ max. deg.; hence
$\sqrt{(DF^2+FE^2)} = \sqrt{81{\cdot}38} = 9$ deg. of 69 miles for DE $= 620$ miles. *Ans.*

6. Here let C denote the position of Cape Town; and as *both* places are in South latitude, let H denote the position of Cape Horn; complete the triangle by drawing the perpendicular from H *towards the Equator.*



Diff. of long. $= 85\frac{1}{2}$ deg. on a parallel of 34°, where a degree of long.

$= 53$ mi. $+ \frac{40-34}{10}$ of 7 mi. $= 57{\cdot}2$ mi.;

hence AC $= \frac{57{\cdot}2}{69} \times 85\frac{1}{2} = 70{\cdot}9$ max. deg.; and we have AH, the diff. of lat., $= 22$ max. deg.; hence $\sqrt{(HA^2 + AC^2)} = \sqrt{5511} = 74{\cdot}2$ deg. of 69 miles for HC $= 5120$ mi. *Ans.*

7. Diff. of long. $63\frac{7}{12}$ deg. on a parallel of $49\frac{1}{2}$ deg., where a degree $= 44$ mi. $+ \frac{50-49\frac{1}{2}}{10}$ of 9 mi. $= 44{\cdot}45$ mi.; hence $44{\cdot}45$ mi. $\times 63\frac{7}{12} = 2830$ mi. *Ans.*

8. In the fig. on p. 76, let D denote the position of Rio, and E that of the Lizard.

Diff. of long. $= 37\frac{4}{15}$ degrees on a parallel of 23° where a degree is $= 60$ mi. $+ \frac{30-23}{10}$ of 5 mi. $= 63\frac{1}{2}$ mi.

$\frac{63\frac{1}{2}}{69} \times 37\frac{4}{15} = 34{\cdot}3$ max. degrees $=$ DF.

$\sqrt{(DF^2 + FE^2)} = \sqrt{6498} = 80{\cdot}6$ deg. of 69 miles for FE $= 5560$ mi. *Ans.*

9. Diff. of long. $= 7\frac{29}{60}$ deg. on a parallel of 45°, where a degree $= 48\frac{3}{4}$ mi. (*see* note 2, p. 72); hence $48\frac{3}{4} \times \frac{449}{60} = 365$ mi. *Ans.*

10. In figure, p. 77, let E represent Suez, F the other place.

Diff. of long. $= 10\frac{5}{12}$ deg. on a parallel of $12\frac{5}{6}$ deg., where a degree is $= 65$ mi. $+ \frac{20-12\frac{5}{6}}{10}$ of 3 mi. $= 67$ mi.

$\frac{67}{69} \times \frac{127}{12} = 10{\cdot}2$ max. deg. for DF.

$\sqrt{(DE^2 + DF^2)} = \sqrt{400} = 20$ deg. of 69 mi. $= 1380$ mi. *Ans.*

11. In the fig. on p. 76, let E denote the position of Mocha, and D that of Buenos Ayres.

Diff. of long. $= 101\frac{5}{6}$ deg. on a parallel of $34\frac{37}{60}$ deg. where, in the usual way, the length of a degree will be found to be $56\frac{3}{4}$ miles.

$\frac{56\frac{3}{4}}{69} \times 101\frac{5}{6} = 83\frac{3}{4}$ max. deg. for DF.

$\sqrt{(DF^2+FE^2)} = \sqrt{9329} = 96{\cdot}59$ deg. of 69 miles $= 6660$ mi. *Ans.*

12. In the fig. on p. 77, let E denote the position of Paris, and F that of Rome.

Diff. of long. $= 10\frac{1}{6}$ deg. on a parallel of $41\frac{13}{15}$ deg., where, in the usual way, the length of a degree is found to be 51·3 mi.

$\frac{51{\cdot}3}{69} \times 10\frac{1}{6} = 7{\cdot}56$ max. deg. for DF.

$\sqrt{(DF^2+DE^2)} = \sqrt{105{\cdot}69} = 10{\cdot}28$ deg. of 69 miles $= 710$ mi. *Ans.*

13. In the fig. on p. 76, let E represent New York, and D Philadelphia.

Diff. of long. $= 1\frac{1}{3}$ deg. on a parallel of 40°,
$= 53$ mi. $\times 1\frac{1}{3} = 70\frac{2}{3}$ mi. $=$ DF; and FE the diff. of lat. is 1 deg. of 69 mi.; hence
$\sqrt{(DF^2+FE^2)} = \sqrt{9754} = 99$ mi. *Ans.*

14. In the fig. on p. 76, let D represent New York, and E Liverpool.

Diff. of long. $= 71$ deg. on a parallel of 41°, where, in the usual way, a degree is found to be 52 mi.

$\frac{52}{69} \times 71 = 53{\cdot}5$ max. deg. for DF.

$\sqrt{(DF^2+FE^2)} = \sqrt{3006} = 54{\cdot}83$ deg. of 69 miles for DE $= 3780$ mi. *Ans.*

EXERCISES 13.

1. New York is farther west by $73\frac{1}{1\frac{1}{2}}$ deg. of 4 min. $=296$ min., or 4 hrs. 56 min. earlier, $=7$ hrs. 4 min. A.M. *1st Ans.*

Petersburg, farther east by $30\frac{1}{6}$ deg. of 4 min. $=121$ min. later $=2$ hrs. 1 min. P.M. *2nd Ans.*

Constantinople, farther east by 29 deg. of 4 min. $=116$ min. later $=1$ hr. 56 min. P.M.

2. Coventry, farther west by 90 deg. of 4 min. $=360$ min. or 4 hours earlier, $=8$ o'clock A.M. *Ans.*

3. Sydney, farther east by $148\frac{1}{6\frac{1}{0}}$ deg. of 4 min. $=594$ min. later $=9$ hrs. 54 min. P.M. *Ans.*

4. Lima, farther west by $133\frac{4}{1\frac{1}{5}}$ deg. of 4 min. $=535$ min., or 8 hrs. 55 min. earlier, $=11$ hrs. 5 min. A.M. *Ans.*

5. Shanghai, farther west by $19\frac{3}{4}$ deg. of 4 min. $=79$ min. or 1 hr. 19 min. earlier, $=4$ hrs. 41 min. A.M. *Ans.*

6. Lisbon, farther west by $40\frac{1}{1\frac{1}{2}}$ deg. of 4 min. $=162$ min. or 2 hrs. 42 min. earlier, $=2$ hrs. 18 min. P.M. *Ans.*

7. New Orleans, farther west by $94\frac{3}{1\frac{1}{5}}$ deg. of 4 min. $=379\frac{1}{1\frac{1}{5}}$ min. or 6 hrs. 19 min. earlier, $=8$ hrs. 41 min. A.M. *Ans.*

8. Noon at sea $=1$ hr. 30 min. later $=90$ min. $\div 4$ min. $=22°\ 30'$ E. *Ans.*

9. Noon is 15 min. earlier; $\therefore$ 15 min. $\div 4$ min. $=3$ hrs. 45 min. farther west; hence $18°\ 27' - 3°\ 45' = 14°\ 42'$ E. *Ans.*

10. Petersburg time, 1 hr. 30 min., is 2 hours later; hence 120 min. $\div 4$ min. $= 30°$ E. *Ans.*

MISCELLANEOUS QUESTIONS.

1. The second exceeds the first by 17; hence $\frac{1}{2}(60-17)$ and $\frac{1}{2}(60+17)=21\frac{1}{2}$ and $38\frac{1}{2}$. *Ans.*

2. The cost at $\frac{1}{8}$ of a penny exceeds the cost at $\frac{1}{10}$ of a penny by $6d.$; hence $6d.\div(\frac{1}{8}-\frac{1}{10})d.=6d.\div\frac{1}{40}d.=240$. *Ans.*

3. A's gain is $\frac{6}{28}$ or $\frac{3}{14}$ of his outlay, B's is $\frac{21}{88}$; ∴ the gains of A and B are as 264 : 294, or as 44 : 49; hence $\frac{5}{44}$ of $100=11\frac{4}{11}$ per cent. *Ans.*

4. Barbadoes, farther west by $59\frac{2}{3}$ deg. of 4 min. $=239$ min. or 3 hrs. 59 min. earlier, $=4$ o'clock P.M. *Ans.*

5. The L.C.M. of 16, 15, and 10 makes the whole work consist of 240 units, of which A does 15 per day, B 16, C 24; together 55 units per day; hence $240\div55=4\frac{4}{11}$ days. *Ans.*

6. $\frac{2}{100}-\frac{2}{102}=\frac{51-50}{51\times50}$ of £340, $=\frac{1}{150}$ of £2 $=2s.\ 8d.$ *Ans.*

7. 12 m. $=17$ w., ∴ 9 m. $=\frac{9}{12}$ of 17 w. $=12\frac{3}{4}$ w.; and 9 m. together with 11 w. $=23\frac{3}{4}$ w.; for which the time would be $\frac{17}{23\frac{3}{4}}$ of $\frac{68}{95}$ of $9\frac{1}{2}$ hrs., $=6\frac{4}{5}$ hrs. *Ans.*

8. Bought 1 for $\frac{2}{3}$ of a penny together with 1 for $\frac{3}{4}$ of a penny, $=2$ for $\frac{17}{12}d.$; and sold the 2 for $\frac{19}{7}d.$; hence, $\frac{19}{7}-\frac{17}{12}$ or $\frac{1}{84}$ of a penny was the gain on 2 oranges, or 1 penny was the gain on 14 dozen; ∴ $4d.$ was the gain on 56 dozen. *Ans.*

9. 47 flor. + £1. $=114s.$, and £47 + 1 flor. $=942s.$; hence, if the total, $1056s.$, be divided between A and B, giving B 10 times as much as A, the share of

A will be $\frac{1}{11}$ of it; or 96*s.*; therefore he must have given to B the value of 114−96 or 18*s.*, that is, 9 flor. *Ans.*

10. Since 9 of A's class do as much per hour as 11 of B's, A can do a piece of work in 9 hours which B can do in 11; hence if the whole of that piece of work consist of 99 units, A can do 11 of these units per hour, and B, 9; together 20 units per hour; therefore the whole in 99÷20=4 hrs. 57 min. *Ans.*

11. To give 45*d.* to each requires 15*d.*+18*d.* or 33*d.* more than to give 42*d.* to each; hence 45−42, or 3*d.* more to each, would amount to 33*d.* more in all; so that the no. of poor=33÷3=11. *Ans.*

12. B has £14+£30, or £44, more than A; hence the sum, B+A, being £87, and the difference, B−A, £44, A has $\frac{1}{2}$ (£87−£44)=£21. 10*s.*, B £65. 10*s.*, and C £35. 10*s.* *Ans.*

13. The 2nd part + the 1st=100
the 2nd part−7 times the 1st= 10

∴ 8 times the 1st= 90,

and hence the parts are $11\frac{1}{4}$ and $88\frac{3}{4}$. *Ans.*

14. 20 apples=9 pears, ∴ 12 apples=$\frac{12}{20}$ of 9 pears=$5\frac{2}{5}$ pears; hence 12 apples with 12 pears=$17\frac{2}{5}$ pears for $14\frac{1}{2}d.$ and, accordingly, 10*d.* will buy $\frac{10}{14\frac{1}{2}}$ of $17\frac{2}{5}$ pears,=$\frac{20}{29}$ of $\frac{87}{5}$=12 pears. *Ans.*

15. London farther west by $13\frac{1}{4}$ deg. of 4 min. =53 minutes earlier than 1 hr. 53 min. A.M., viz. 1 o'clock A.M. *Ans.*

16. The no. of men and children together=$\frac{3}{5}$ of 1000=600, to be divided in the proportion of 3 : 1;

hence $\frac{3}{4}$ of 600 = 450 men, $\frac{2}{5}$ of 1000 = 400 women, 600 − 450 = 150 children. *Ans.*

17. If the whole work consist of the L.C.M. of 21 and 49, viz. 147 units, A does in 1 day $147 \div 5\frac{1}{4}$ or 28 of these, B $147 \div 6\frac{1}{8}$ or 24, A, B, and C $147 \div 2\frac{1}{10} = 70$; hence C does $70 - 28 - 24 = 18$ units per day, and therefore the whole in $147 \div 18$ or $8\frac{1}{6}$ days. *Ans.*

18. $\frac{1}{12}$ of $5 = 2\frac{1}{12}$ per cent.; hence the interest on any sum is $\frac{2\frac{1}{12}}{100}$, or $\frac{7}{240}$ of the sum, and the discount on the same sum is $\frac{2\frac{1}{12}}{102\frac{1}{12}}$ or $\frac{7}{247}$ of the sum; hence 7 times $(\frac{1}{240} - \frac{1}{247}) = \frac{49}{59280}$. *Ans.*

19. At six o'clock, the hour hand, H, is 30 minute spaces in advance of the minute hand, M; and an angle of 60° is subtended by an arc of 10 minute spaces. Hence the hands will be 10 spaces apart when M gains $30 - 10$, or 20 spaces, on H, and again when M gains $30 + 10$, or 40 spaces. Therefore, as M gains 55 spaces in an hour, we have $\frac{20}{55}$ and $\frac{40}{55}$ of 60 minutes $= 21\frac{9}{11}$ and $43\frac{7}{11}$ minutes past six. *Ans.*

20. 1 boy's ability is equal to $\frac{9}{21}$ or $\frac{3}{7}$ of a man's; 1 woman's is equal to $\frac{9}{14}$ of a man's;
hence 1 man, 1 woman, and 1 boy, together, exert $1 + \frac{3}{7} + \frac{9}{14} = 2\frac{1}{14}$ of a man's ability; and therefore if 9 times a man's ability require $7\frac{1}{4}$ hours, $2\frac{1}{14}$ of a man's ability would require $\frac{9}{2\frac{1}{14}}$ or $\frac{126}{29}$ of $7\frac{1}{4}$ hours $= 31\frac{1}{2}$ hrs. *Ans.*

21. When the debt is paid, M and N will, of course, still have between them £56. 4*s.* + £54. 6*s.*, but in portions as 3 : 2. So M will then have $\frac{3}{5}$ of

£110. 10*s.* = £66. 6*s.*; which shows that N's debt was = £66. 6*s.* − £56. 4*s* = £10. 2*s.* *Ans.*

22. Ispahan farther east by $39\frac{7}{15}$ deg. of 4 min. = 158 min. or 2 hrs. 38 min. later, = 6 hrs. 38 min. A.M. *Ans.*

23. The no. of boys = $\frac{1}{1\frac{1}{3}}$ or $\frac{3}{4}$ of the no. of women, and the no. of women = $1\frac{1}{2}$ of the no. of men; hence the nos. of men, women, and boys are as 1, $1\frac{1}{2}$, and $\frac{3}{4}$ of $1\frac{1}{2}$, or as 8 : 12 : 9; and therefore the amounts given to the whole of each class are as 5*s.* 6*d.* × 8, 3*s.* 6*d.* × 12, and 1*s.* 6*d.* × 9, or as 44, 42, and $13\frac{1}{2}$, or as 88 : 84 : 27; sum 199.

Accordingly, £9. 19*s.* ÷ 199, or 1*s.*, multiplied by 88, 84, 27, would give the amounts paid to each class. Thus the no. of persons in each class = 88*s.* ÷ $5\frac{1}{2}$*s.*, 84*s.* ÷ $3\frac{1}{2}$*s.*, and 27*s.* ÷ $1\frac{1}{2}$*s.*, or 16 men, 24 women, 18 boys. *Ans.*

24. $66\frac{1}{2}$ degrees is between 60° of $34\frac{1}{2}$ miles and 70° of $23\frac{1}{2}$ miles; hence the length required = $23\frac{1}{2}$ mi. + $\frac{70-66\frac{1}{2}}{10}$ of 11 mi. = 27·4 miles. *Ans.*

25. The son gets $\frac{4}{9}$, his sister $\frac{1}{4}$ of $\frac{5}{9}$ or $\frac{5}{36}$; together = $\frac{7}{12}$; therefore the widow gets $\frac{5}{12}$; and hence $\frac{4}{9} - \frac{5}{12}$ or $\frac{1}{36}$ of the property = £150; ∴ the sister's share, $\frac{5}{36}$, = £150 × 5 = £750. *Ans.*

26. The sum of their rates per hour = $25\frac{1}{2}$ mi. ÷ 3 = $8\frac{1}{2}$ mi., and the difference = 6 mi. ÷ 4 = $1\frac{1}{2}$ mi.; hence their rates are = $\frac{1}{2}$ ($8\frac{1}{2} \pm 1\frac{1}{2}$), or 5 mi. and $3\frac{1}{2}$ mi. *Ans.*

27. The original shares of A, B, and C were as 8 : 5 : 3, that is, A had $\frac{8}{16}$ of the estate, B $\frac{5}{16}$, C $\frac{3}{16}$. When A sold half his share to C, then A had $\frac{4}{16}$, B $\frac{5}{16}$,

C $\frac{7}{16}$, and thus B and C together had $\frac{12}{16}$; but now C sold 100 acres to B, and made himself equal with B, and thus B had $\frac{6}{16}$ and C $\frac{6}{16}$; therefore, either $\frac{6}{16}-\frac{5}{16}$, or $\frac{7}{16}-\frac{6}{16}=\frac{1}{16}$ of the estate was 100 acres:
hence A's original $\frac{8}{16}=800$ acres, B's $\frac{5}{16}=500$, C's $\frac{3}{16}=300$. *Ans.*

28. 9 times the less + 9 times the greater = 360,
9 „ „ −5 „ „ = 33;
∴ 14 times the greater = 322,
and hence the greater is 23, and the less 17. *Ans.*

29. $58\frac{2}{5}$ degrees is between 50 deg. of 44 miles and 60 deg. of $34\frac{1}{2}$ miles; hence

$$34\tfrac{1}{2}+\frac{60-58\frac{2}{5}}{10} \text{ of } (44 \text{ mi.}-34\tfrac{1}{2} \text{ mi.})=36 \text{ mi.}$$ *Ans.*

30. M had gone $\frac{1}{3}$ of 8 miles when N started; hence N gained $2\frac{2}{3}$ miles in $2\frac{1}{3}$ hours, $=1\frac{1}{7}$ mi. per hour;
∴ N's rate $=8+1\frac{1}{7}=9\frac{1}{7}$ miles an hour. *Ans.*

31. At 7 o'clock, the hour hand H is 35 minute spaces in advance of the minute hand, M; therefore, the interval between them will be equal to 12 spaces when M has gained on H 35 − 12, or 23 spaces, and again when M has gained 35 + 12, or 47 spaces; hence, as M's rate of gain is 55 spaces per hour, we have $\frac{23}{55}$ and $\frac{47}{55}$ of 60 minutes $=25\frac{1}{11}$ and $51\frac{3}{11}$ min. past 7. *Ans.*

32. We have 23 times the less + 31 times the greater = 8800 yds.,
also 23 times the less + 23 times the greater = 7268 yds.
∴ 8 times the greater = 1532;
hence the greater is $191\frac{1}{2}$ yds., and the less $124\frac{1}{2}$. *Ans.*

33. One son got $\frac{5}{12}$, the other $\frac{7}{24}$, the widow $\frac{7}{24}$; hence $\frac{5}{12}-\frac{7}{24}$ or $\frac{1}{8}$ of the estate was £784, and therefore $\frac{7}{24}$ of it was $\frac{7}{3}$ of £784 = £1829. 6*s*. 8*d*. *Ans.*

34. Take 2457, a common multiple of 27, 117, 91, and 63, as the no. of units in the whole work.

A does 91 of these units per day, B 84, C 81, D 78; in all, 334 units per day; hence $2457 \div 334 = 7\frac{119}{334}$ days. *Ans.*

35. Y £640 for 5 months = £3200 for 1 month,
480 for 7 „ = 3360 „
6560

Z £500 for 6 months = £3000 for 1 month,
1000 for 6 „ = 6000 „
9000
15560.

Hence the shares are:—

$\frac{656}{1556}$ of $\frac{£1945}{3} = \frac{5}{12}$ of £656 = £273$\frac{1}{3}$ to Y,
$\frac{5}{12}$ of 900 = 375 to Z. } *Ans.*

36. £650 × R = £676; ∴ R = 676 ÷ 650 = 52 ÷ 50 = 1·04; and hence the sum = £650 ÷ 1·04 = £625. *Ans.*

37. The product of the sum and difference of hypotenuse and perpendicular = $28^2 = 784$; hence their difference = 784 ÷ 98 = 8, and the perpendicular = $\frac{1}{2}(98-8)$ or 45; so that the area is = $\frac{1}{2}(28 \times 45) = 630$ sq. in. *Ans.*

38. The son's present age is equal to $\frac{5}{12}$ or $\frac{10}{24}$ of the father's, and the daughter's is equal to $\frac{3}{8}$ or $\frac{9}{24}$ of the father's; therefore the ages of father, son, and daughter are as 24 : 10 : 9; moreover, the ages of the son and daughter are together $11\frac{1}{4}$ less than the father's; but the ages of son and daughter are $\frac{19}{24}$ of

the father's, and hence $\frac{5}{24}$ of the father's age is equal to $11\frac{1}{4}$ years, or the father's is $\frac{24}{5}$ of $11\frac{1}{4}=54$, the son's is $\frac{5}{12}$ of $54=22\frac{1}{2}$, and the daughter's is $\frac{3}{8}$ of $54=20\frac{1}{4}$. *Ans.*

39. The L.C.M. of 18, 20, and 45 is 180, and this may be conveniently regarded as equal parts or units of which the whole work consists. A does 10 of these in a day, B 9, C 8; sum, 27. Now 9×2, or 18 such units would have been added to the work, if B had begun with the others, and then $180+18$, or 198 units would have been done at the rate of 27 units per day; hence $198\div27=7\frac{1}{3}$ days. *Ans.*

40. 400 gallons = 110900 cubic inches per minute $=240^2\times\frac{11}{14}\times$ the number of inches rise per minute; hence the rise in inches per minute $=1109\div(576\times\frac{11}{14})$ $=1109\times7\div3168$, or 2·45. *Ans.*

41. In 10 seconds he walks $\frac{1}{360}$ of $1760\times3\frac{3}{4}=18\frac{1}{3}$ yards; and in the same time the train runs $90+18\frac{1}{3}$ or $108\frac{1}{3}$ yards = 650 yards per minute,

$$\frac{650\times60}{1760}=\frac{325\times3}{44},$$ or $22\frac{7}{44}$ miles an hour. *Ans.*

42. Two o'clock P.M. is $4\frac{1}{2}$ hrs. or 270 min. later; and therefore the required longitude $=270\div4$ or $67\frac{1}{2}$ degrees farther east than that of New York, viz. in 6° 25′ W. *Ans.*

43. When the selling price is diminished by $10\frac{5}{7}$ per cent., there is left $89\frac{2}{7}$ per cent. of the selling price equal to the prime cost; and therefore for £100 of prime cost the selling price is $\frac{100}{89\frac{2}{7}}$ of £100 $=\frac{28}{25}$ of £100 = £112, showing a gain of 12 per cent. *Ans.*

44. To do the whole singly, B's time would be equal

to $\frac{3}{4}$ of A's, C's time to $\frac{5}{4}$ of B's, and D's time to $\frac{5}{6}$ of C's; but A's time is 60 hrs.; therefore B's, $=\frac{3}{4}$ of 60, is 45 hrs., C's $=\frac{5}{4}$ of 45, is $56\frac{1}{4}$ hrs., D's $=\frac{5}{6}$ of $56\frac{1}{4}$, is $46\frac{7}{8}$ hrs. 1*st Ans.*

Now take the L.C.M. of 60, 45, 225, and 375, viz. 4500, for a no. of equal portions or units of which the whole work consists:—A would do 75 of these units per hour, B 100, C 80, D 96; sum, 351;

hence $4500 \div 351 = 12\frac{3}{3}\frac{2}{9}$ hrs. 2*nd Ans.*

45. The first and second quantities were sold at $\frac{5}{6}$ of a penny and $\frac{13}{16}$ of a penny respectively:
hence $\frac{13}{16}$ of the second $-\frac{5}{6}$ of the 1st $=4$,
but $\frac{13}{16}$ of the second $+\frac{13}{16}$ of the 1st $=\frac{13}{16}$ of 90;
$\therefore$ $(\frac{13}{16}+\frac{5}{6})$ of the first quantity $=\frac{13}{16}$ of $90-4$, or $\frac{79}{48}$ of the first $=69\frac{1}{3}$, or 79 times the first $=553\times 6$;
hence the first, $=7\times 6$, is 42, and the second is 48.
Ans.

46. The no. of women is equal to the no. of men $+6$, and the no. of boys is equal to the no. of men $+10$.

Accordingly, if at the beginning 6 women received each 3*s.* 10*d.* $=$ 23*s.*, and 10 boys each 1*s.* 8*d.* $=$ 16*s.* 8*d.*, there would remain £5. 11*s.* 8*d.* to be shared in the proportion of 68 : 46 : 20, or as 34 : 23 : 10, among three classes, men, women, and boys, equal in number; and thus the sum taken by the whole no. of men would be equal to $\frac{34}{67}$ of 111*s.* 8*d.*

$$= \frac{335s. \times 34}{67 \times 3} = \frac{1}{3} \text{ of } 170s.,$$

or 56*s.* 8*d.*, which, at 5*s.* 8*d.* for each man, quotes 10 as the no. of men, and hence 16 women, and 20 boys. *Ans.*

47. Let 6, 12, and 18 represent the capitals of A, B, C.

A invests 4 for 5 months = 36 for 1 month,
3 for 3 " = 9 "
6 for 3 " = 18 "
63

B invests 12 for 4 months = 48 for 1 month,
8 for 8 " = 64 "
112

C invests 18 for 6 months = 108 for 1 month,
26 for 6 " = 156 "
264

£658½ ÷ 439 = £1½;

hence £1½ multiplied by the distributives makes A £94½, B £168, C £396. *Ans.*

48. Take 180 units, the L.C.M. of 10, 9, and 12, for the whole work. A does 18 of these per day, B 20, C 15; 53 units together per day. If A and B had wrought the whole of C's time, they would have done $18 \times 3\frac{3}{5}$ and $20 \times 2\frac{3}{5} = 116\frac{4}{5}$ units more than sufficient, making the whole amount of work $= 180 + 116\frac{4}{5}$, or $296\frac{4}{5}$ units, done at the rate of 53 units per day, viz. in $1484 \div 265$ or $5\frac{3}{5}$ days. *Ans.*

49. The section area of the reservoir, in square inches, is equal to $144 \times 9 \times 187 \times 84$, and that of the pipe is equal to $14^2 \times \frac{11}{14} = 154$; also, the no. of cubic inches of water delivered per minute is $= \frac{1}{60}$ of 154 sq. in. $\times$ 36 in. $\times$ 1760 $\times$ 10; hence the time in which an inch of depth, or $144 \times 9 \times 187 \times 84$ cubic inches will be delivered is $= \frac{144 \times 9 \times 187 \times 84 \times 60}{154 \times 36 \times 1760 \times 10}$ min. $= \frac{9 \times 9 \times 17}{110}$ $= 12$ min. $31\frac{11}{11}$ sec. *Ans.*

50. Each man did $\frac{1}{7} \div 20$, or $\frac{1}{140}$ of the work in 1 day; ∴ in the 25 days the original no. of men did

$\frac{2}{3}\frac{5}{8}$; and hence the three additional men did the remaining $\frac{3}{2}\frac{3}{8}$ in 5 days, and could have done $\frac{3}{7}$ in 20 days; hence $\frac{3}{7} : \frac{5}{7} :: 3$ men : 5 men. *Ans.*

51. Write T for the no. of pounds of tea, and C for that of coffee at 17*d.*; then, since the amounts of the sales were equal, we have $C = \frac{3}{1}\frac{9}{7}$ of T, and therefore $C - T = \frac{3}{1}\frac{9}{7}$ of $T - T = \frac{2}{1}\frac{2}{7}$ of $T = 5\frac{1}{2}$ lb.; hence $4T = 17$ lb., there were accordingly $4\frac{1}{4}$ lb. of tea and $9\frac{3}{4}$ lb. of coffee. *Ans.*

52. In the fig. p. 76, let D be the position of Gibraltar, E that of Venice. The diff. of longitude is $17\frac{2}{3}$ degrees on the parallel of 36° 7′, where the length of a degree is $= 53$ mi. $+ \frac{40 - 36\frac{7}{60}}{10}$ of $(60 - 53)$ mi. $= 55{\cdot}72$ miles, $= \frac{55{\cdot}72}{69} \times \frac{53}{3} = \frac{214}{15}$ maximum degrees; and as FE, the diff. of latitude, is $9\frac{13}{60}$ or $\frac{553}{60}$ max. deg. we have $DF^2 + FE^2 = \frac{1}{3600}$ of 1038545, the root of which is $= \frac{1}{60}$ of $1019{\cdot}09 = 16{\cdot}985$ deg. of 69 mi. $= 1170$ miles. *Ans.*

53. Here we are to find how many dozen at 26*s.* 9*d.* and how many at 17*s.* $6\frac{3}{4}d.$ will together pay the cost of 63 dozen at 25*s.* The quantities will be as $300d.$ $- 210\frac{3}{4}d.$ is to $321d. - 300d.$ or as $89\frac{1}{4} : 21$, or as $17 : 4$; hence $\frac{17}{21}$ of $63 = 51$ doz. *Ans.*

54. The train from London ran 30 miles in 90 minutes, and therefore in 35 min. had run $11\frac{2}{3}$ miles, and was then $18\frac{1}{3}$ miles from the place of meeting; hence the train from London covered $18\frac{1}{3}$ miles in the time the other train covered 20 miles; therefore the rate of the former was to that of the latter as $18\frac{1}{3} : 20$, or as $11 : 12$. *Ans.*

55. The required sum $\times (1{\cdot}05^3 - 1{\cdot}15)$ is to be equal to £13·34375; and as $1{\cdot}05^3 = 1{\cdot}157625$, the required sum is = £13·34375 ÷ ·007625 = £106·75 ÷ ·061 = £1750. *Ans.*

56. Take the L.C.M. of 32, 28, 24, and 21, viz. 672, for the units of which the whole work may be supposed to consist. In 1 hour A can do 21 of these, B 24, C 28, D 32; sum, 105 units. Now, supposing the hours of absence filled up, this would add 21 + 28 units to the 672 units, making 721 units, which the four together could have done in the time the work was actually done; hence $721 \div 105 = 6\frac{1}{15}$ hrs. *Ans.*

57. The joint stock being £1534, we have A's stock £456, B's £546, C's the remaining £532.

(*i*) To obtain A's gain: £532 gains £108·8 in 8 months, therefore the gain on £456 for 7 months would be

$$= \frac{£108{\cdot}8 \times 456 \times 7}{532 \times 8} = \frac{£1088 \times 24}{40 \times 8} = £81.\ 12s.\ 1st\ Ans.$$

(*ii*) To obtain B's time:—£532 gains £108·8 in 8 months, therefore the time for £546 to gain £132·6 would be

$$= \frac{8\text{ mo.} \times 532 \times 132{\cdot}6}{546 \times 108{\cdot}8} = \frac{6\text{ mo.} \times 532 \times 17}{42 \times 136} = 9\tfrac{1}{2}\text{ mths.}$$

2nd Ans.

58. The amount of 11 women at 1*s.* 6*d.* exceeds that of 2 men at 2*s.* 3*d.* by 12*s.*; hence if we write M for the no. of men, and W for the no. of women, we have:—

$$2\tfrac{1}{4}\,M - 1\tfrac{1}{2}\,W = 12,$$

$$\text{and } 2\tfrac{1}{4}\,M + 1\tfrac{1}{2}\,W = 42;$$

$$\therefore\ 4\tfrac{1}{2} \text{ times the no. of men is } 54;$$

hence there were 12 men and 10 women. *Ans.*

59. Length of 1st train $= 40 + 32 \times 12 + 5 \times 12$ ft.,
„ 2nd „ $= 40 + 32 \times 17 + 5 \times 17$ ft.;
together $= 80 + 37 \times 29 = 1153$ ft.

Now, $40 + 30 = 70$ miles an hour $= \frac{308}{3}$ feet per second; hence $1153 \div \frac{308}{3} = \frac{3459}{308} = 11\ \frac{71}{308}$ secs. *Ans.*

60. Since A, B, and C together do $\frac{1}{5}$ of the work per day, and A's portion is equal to $\frac{5}{7}$ of the joint work of B and C per day, let $\frac{1}{5}$ be divided into two parts as $1 : \frac{5}{7}$, or $7 : 5$; then A's portion per day is $= \frac{5}{12}$ of $\frac{1}{5} = \frac{1}{12}$.

Again: Since C's portion is equal to $\frac{1}{3}$ of the joint work of A and B, then $\frac{1}{5}$ divided into two parts as $1 : \frac{1}{3}$, or $3 : 1$, gives C's portion per day $= \frac{1}{4}$ of $\frac{1}{5} = \frac{1}{20}$; therefore B's portion per day $= \frac{1}{5} - (\frac{1}{12} + \frac{1}{20}) = \frac{1}{15}$.

Hence the whole work could be done by A alone in 12 days, by B in 15, and by C in 20. *Ans.*

61. The difference of the ages is, of course, always the same. That difference is now equal to $\frac{5}{12}$ of the father's present age; and 24 years ago it was equal to $\frac{3}{4}$ of (the father's present age -24), that is, equal to $\frac{3}{4}$ of the father's age -18.

Hence $\frac{3}{4} - \frac{5}{12}$, or $\frac{1}{3}$ of the father's present age $= 18$; so that the father's present age is 54, and the son's $31\frac{1}{2}$.

Again: The future ages are to be as $8 : 5$, and therefore the difference of these ages will be equal to $\frac{3}{8}$ of the greater; but the difference will then be, as now, $= 54 - 31\frac{1}{2}$, or $22\frac{1}{2}$ years, and if $\frac{3}{8}$ of the father's age will be $22\frac{1}{2}$, his age will be 60, which is 6 years hence. *Ans.*

62. First income $= £3\frac{1}{2} \times 36\frac{1}{2}$ cents. $= £127.\ 15s.$; second income $= £156\frac{3}{5}$.

Cash got by selling out = £87 × 36½; and this divided by the price of the 4½ per cent. stock must give the no. of cents. purchased in that stock, which is equal to £156⅜ ÷ 4½;

$$\text{hence, } \frac{£87 \times 36\frac{1}{2} \times 4\frac{1}{2}}{£156\frac{3}{8}} = \frac{87 \times 73 \times 9 \times 5}{783 \times 4} = \frac{365}{4} = 91\frac{1}{4}.$$

Ans.

63. (21*s.* + 10*s.*) multiplied by the no. of guineas, or of half-sovereigns, amounts to 4650*s.*; hence there are 150 of each denomination, amounting to £157. 10*s.* and £75.

$$\frac{4\frac{1}{2}}{100} \text{ of } £157\frac{1}{2} = £6.\ 6s.$$

$$\frac{4\frac{2}{3}}{100} \text{ of } £75 = £3.\ 10s.$$

sum of the interests for 1 year = £9. 16*s.*;
sum for the whole period = £239. 17*s.* − £232. 10*s.*;
hence, £7. 7*s.* ÷ £9. 16*s.* = 147 ÷ 196 = ¾ yr., or 9 months

Ans.

64. We are here to divide 70*s.* into two parts so that 125 times the first shall be equal to 85 times the second, or that the first shall be to the second as 17 : 25; hence $\frac{17}{42}$ of 70*s.* = 28*s.* 4*d.* for the first, and, of course, 70*s.* − 28*s.* 4*d.*, or 41*s.* 8*d.*, for the second. *Ans.*

65. The stocks of A and B are as 8 : 9, and their gains as 7 : 6; and their times are as the quotients of their gains by their stocks, that is, as $\frac{7}{8} : \frac{6}{9}$, or as 21 : 16; hence B's time = $\frac{16}{21}$ of 10½ months = 8 months.

Ans.

66. Call the common result R; then the successive parts are equal to R − 12, R + 19, $\frac{1}{12}$ of R, and 19 times R; sum, $21\frac{1}{12}$ times R + 7 = 536; hence R = 529 ÷ $21\frac{1}{12}$ = $25\frac{1}{11}$; and hence the parts are $13\frac{1}{11}$, $44\frac{1}{11}$, $2\frac{1}{11}$, $476\frac{8}{11}$. *Ans.*

67. The product of the first part by $3\frac{1}{2} \times 5$ is to be equal to that of the second by 4×7, and to that of the third by $3\frac{1}{4} \times 8$; accordingly the parts will be as $\frac{2}{35} : \frac{1}{28} : \frac{1}{26}$, or as 104 : 65 : 70; sum, 239; and the distributives multiplied by $836\frac{1}{2} \div 239$, or by $3\frac{1}{2}$, give £366, £227½, £245. *Ans.*

68. The costs, successively, were as 100, 105, 1·06 of 105, and 1·07 of 1·06 of 105, or as 20, 21, $1{\cdot}06 \times 21$, and $1{\cdot}07 \times 1{\cdot}06 \times 21$; the first and last of these are as 10 : 11·9091; hence A's prime cost $= \frac{100000}{119091}$ of £1984·85 = £28355000 ÷ 17013 = £1666⅔. *Ans.*

69. The original strength is $\frac{83}{92}$; and this fraction is to have its terms equally increased so that the value of the resulting fraction shall be $\frac{23}{25}$;

hence 25 − 23 : 23 :: 92 − 83 : 103½, the increased numerator; ∴ 103½ − 83 = 20½ pints. *Ans.*

70. $\frac{1}{12} \times \frac{160}{100} \times \frac{17}{40}$ of the original sum = £102, or $\frac{1}{3} \times \frac{1}{100}$ of it = £6; hence the original sum is £1800. *Ans.*

71. The sum of the lengths = the sum of the rates per second × 4, and also = the difference of ditto × 20; hence the sum of the rates is to their difference as 20 : 4, or as 5 : 1, and therefore the rate of the engine and tender is to that of the train as 5 + 1 : 5 − 1, or as 3 : 2; and, accordingly, the rate of the train is $= \frac{2}{3}$ of 27 = 18 miles. Hence the sum of the rates per second in feet $= 45 \times \frac{5280}{3600} = 66$ feet; and therefore the sum of the lengths = 66 ft. × 4, and the length of the train 264 − 25, or 239 feet. *Ans.*

72. Let C signify the cost of a cow in pounds, S that of a sheep: we have $16C \times 1{\cdot}075 + 120S \times 1{\cdot}06$ = £496·5,

$$\text{or } 16C \times \frac{1075}{1060} + 120S = \frac{£49650}{106};$$

$$\text{but } 16C \quad + 120S = £465;$$

$$\therefore 16C \times \frac{15}{106} = \frac{£49650 - £49290}{106},$$

or $16C \times 1\frac{1}{2} = £360$; so that the cost of a cow was £15, and that of a sheep is now readily found to be 37*s.* 6*d.* *Ans.*

73. The product of the greatest by 6×2, that of the smallest by 4×4, and that of the third by 5×3, are to be equal; or these three sums are to be as $\frac{1}{12}$, $\frac{1}{16}$, and $\frac{1}{15}$, or as 20 : 15 : 16; hence the greatest will exceed the smallest by $\frac{5}{20}$ or $\frac{1}{4}$ of the greatest; $\therefore$ the greatest, $= £47\frac{1}{2} \times 4$, is £190; the smallest, $= \frac{3}{4}$ of £190, is £142$\frac{1}{2}$, and the third, $= \frac{4}{5}$ of 190, is £152. *Ans.*

74. The sum of the larger and smaller diameters multiplied by their difference, 44 yards, and by $\frac{11}{14}$ = 4840 sq. yds. $\times$ 12; hence the sum of the diameters $= 110 \times 12 \div \frac{11}{14} = 1680$ yds.; so that the smaller diameter $= \frac{1}{2}(1680 - 44) = 818$ yds. *Ans.*

75. Let R signify the common result; then the successive parts are equal to R-55, R$+44$, $\frac{1}{33}$ of R, and 22 times R; in all, $24\frac{1}{33}$ R$-11 = 1880$; $\therefore$ 793 times R $= 1891 \times 33$, or 13 times R $= 31 \times 33 = 1023$.

Hence R is $78\frac{9}{13}$ and the parts are $23\frac{9}{13}$, $122\frac{9}{13}$, $2\frac{5}{13}$, $1731\frac{3}{13}$. *Ans.*

76. The first $\times \frac{14}{114}$ is to be equal to the second $\times \frac{15}{115}$; therefore the first is to the second as $\frac{57}{7} : \frac{23}{3}$, or as 171 : 161; hence the first, $= \frac{171}{332}$ of £1660, is £855, and the second is £805. *Ans.*

77. The interest on £304. 3*s*. 4*d*. comes to £20. 1*s*. 6*d*.; hence $\frac{401\frac{1}{2}}{6083\frac{1}{3}}$ of $100 = \frac{3}{5}^3 = 6\frac{3}{5}$, the interest on 100; therefore $2\frac{1}{5}$ is the interest on 100 for a third of the time; and the discount is $= \frac{2\frac{1}{5}}{102\frac{1}{5}}$ or $\frac{11}{511}$ of £324. 4*s*. 10*d*. = 12*s*. $8\frac{2}{7}d. \times 11 =$ £6. 19*s*. $7\frac{1}{7}d.$ *Ans*.

78. Let S signify the whole cost of salmon in pence, and C that of cod; then we have—

$$1\frac{1}{5}S + \frac{19}{20}C = 3857\frac{1}{2}d.$$

but $\frac{19}{20}S + \frac{19}{20}C = \frac{19}{20}$ of $3350d. = 3182\frac{1}{2}d.$

$$\therefore \frac{1}{4}S = 675d., \text{ or } S = 2700d.$$

$$3350 - 2700 = C = 650;$$

hence $2700 \div 18$, and $650 \div 5$, quote 150 lb. of salmon and 130 lb. of cod. *Ans*.

79. The proportions of cost for the manufacturer, wholesale dealer, and shopkeeper are as 100, 120, and 1·25 of 120, or as 10 : 12 : 15.

The article cost the shopkeeper $\frac{100}{130}$ of $17\frac{1}{3}s.$, and therefore cost the manufacturer $\frac{10}{15}$ of $\frac{10}{13}$ of $\frac{52}{3}s. = \frac{2}{3}$ of $\frac{40}{3}s. = 80s. \div 9 = 8s.\ 10\frac{2}{3}d.$ *Ans*.

80. It is convenient to assume the whole work to consist of 5985 units, viz. twice the L.C.M. of $21\frac{3}{8}$, $22\frac{1}{6}$, $23\frac{3}{4}$, $26\frac{1}{4}$. A does 280 of the units per hour, B 270, C 252, D 228; sum, 1030 units per hour.

Now, had C worked for the two hours, he would have added 252×2 units to the 5985, making 6489 units to have been done by the four in the time in which the necessary work was actually done; $\therefore 6489 \div 1030 = 6\frac{3}{10}$ hrs. *Ans*.

81. The interest on the present value for 2 months is equal to 26*s*. 3*d*. − 25*s*. 10*d*. = 5*d*., which is therefore

$\frac{1}{6}$ of 10 p.c. $= 1\frac{2}{3}$ per cent. on the present value, and therefore the present value $= \frac{5d. \times 100}{1\frac{2}{3}} = 300d.$ or $25s.$; and the prime cost at present $= \frac{100}{112\frac{1}{2}}$ of $25s.$; hence the prime cost 2 months ago would have been $\frac{100}{101\frac{2}{3}}$ of $\frac{100}{112\frac{1}{2}}$ of $25s. = \frac{6}{6}\frac{0}{1}$ of $\frac{8}{9}$ of $25s. = 21s.\ 10\frac{18}{61}d.$ *Ans.*

82. We are here to divide £5100 into two parts, such that the first multiplied by $\frac{3}{85\frac{1}{2}}$ together with the second multiplied by $\frac{3\frac{1}{2}}{98}$ shall make up £180, or $\frac{2}{5}\frac{}{7}$ of the first with $\frac{1}{28}$ of the second is to equal $\frac{180}{5100}$ or $\frac{3}{85}$ of both.

Hence the 1st is to the 2nd as $\frac{1}{28} - \frac{3}{85} : \frac{3}{85} - \frac{2}{57}$, or as $\frac{85}{84} - 1 : 1 - \frac{170}{171}$, or as $\frac{1}{84} : \frac{1}{171}$, or as 57 : 28, and therefore the 1st part, $= \frac{57}{85}$ of £5100, is £3420, and the 2nd is £1680. *Ans.*

83. The first sum multiplied by 110 is to be equal to the second multiplied by $116\frac{2}{3}$, and to the third multiplied by $112\frac{3}{4}$. These multipliers are as 1320, 1400, and 1353, and hence the sums will be as $\frac{1}{1320}$, $\frac{1}{1400}$, and $\frac{1}{1353}$, or as 4305 : 4059 : 4200; sum 12564.

Accordingly, $\frac{£5235}{12564}$, or $\frac{£5}{12}$, multiplied by the distributives gives £1793$\frac{3}{4}$, £1691$\frac{1}{4}$, and £1750. *Ans.*

84. Length × breadth = 57, and depth × breadth = 19; ∴ the depth is equal to $\frac{1}{3}$ of the length.

Again: Length × depth = 27, that is, the length multiplied by $\frac{1}{3}$ of the length = 27, which means that the square of the length is 81; and hence the length is 9 feet, the breadth $6\frac{1}{3}$, and the depth 3. *Ans.*

85. The difference of the squares on the diameters of the lamina and inner circle $=\frac{14}{11}$ of 55 sq. in. $=70$ sq. in.; hence $10{\cdot}5^2-70=40{\cdot}22=$the square on the diameter of the inner circle; hence that diameter $=\sqrt{40{\cdot}22}=6{\cdot}34$ inches, and the breadth of the ring $=\frac{1}{2}(10{\cdot}5-6{\cdot}34)=2{\cdot}08$ in. *Ans.*

86. The proper speed was to the actual rate as 7 mi. : 5 mi. Then, at 7 miles an hour a mile would have been run in $\frac{1}{7}$ of an hour, and at 5 miles an hour it would have been run in $\frac{1}{5}$ of an hour; hence $\frac{1}{5}-\frac{1}{7}$: $\frac{1}{7}$ :: 36 min. : 90 min., or $1\frac{1}{2}$ hour. *Ans.*

87. The quantity of spirits left each time was lessened according to a uniform ratio, until at the fourth time the remainder was $30\frac{3}{8}$ gallons; therefore the 4th root of $\frac{30\frac{3}{8}}{96}$, or of $\frac{243}{768}$, or of $\frac{81}{256}=\frac{3}{4}$ of the 96 gallons left the first time; and hence $\frac{1}{4}$ or 24 gallons were drawn off each time. *Ans.*

88. $\frac{8}{9}$ of the former allowance at $\frac{120}{100}$ or $\frac{6}{5}$ of the former price $=\frac{16}{15}$; hence $\frac{16}{15}$ of the former yearly charge $=£597$; $\therefore$ $£597-\frac{1}{16}$ of $£597=£559.\ 13s.\ 9d.$ *Ans.*

89.

B's present claim, $91\frac{7}{8}$ gallons at $10s.\ 3\frac{1}{2}d.=11346\frac{9}{16}d.$
A's claim, 245 yards at $3s.\ 9\frac{3}{4}d.$ $=11208\frac{3}{4}d.$
The difference, $137\frac{13}{16}d.$, is 3 months' interest, at 5 per cent. per annum, on the present value of each debt. Hence the present value of each debt is

$$= 137\tfrac{13}{16}d. \times \frac{100}{1\frac{1}{4}}=\frac{2205}{16}\times 80=11025d.$$

Now, to find the term of credit allowed by A is to find the time in which the interest of this present

value will be equal to $11208\frac{3}{4}d. - 11025d. = 183\frac{3}{4}d.$; hence $\frac{183\frac{3}{4} \times 100}{11025 \times 5}$ of 12 months $= \frac{1}{3}$ of 12 months, or 4 months' credit allowed by A, 7 months' by B. *Ans.*

90. The largest $\times 110\frac{1}{2}$ is to be equal to the least $\times 130$, and to the third $\times 117$; hence the three sums are as $\frac{2}{221} : \frac{1}{130} : \frac{1}{117}$, or as 180 : 153 : 170; and therefore £108 is equal to $\frac{27}{180}$ or $\frac{3}{20}$ of the largest sum; hence the largest is £720, the least £612, and the third £680. *Ans.*

91.

$$\text{First, } 28 \text{ hrs.} \times \frac{36 \text{ m.}}{40 \text{ m.}} \times \frac{69 \text{ un.}}{52\frac{1}{2} \text{ un.}} \times \frac{35 \text{ hrs.}}{42 \text{ hrs.}} = 27\frac{3}{5} \text{ hrs.}$$

We have thus found that the 40 superior workmen could do the 69 units in $27\frac{3}{5}$ hours = the work of 1 man for 1104 hrs.

But the work is to be done by 40 men working 2 days and 27 men working 3 days, = the work of 1 man for 80 + 81 or 161 days; ∴ 161 days = 1104 hours, which is $6\frac{6}{7}$ hours per day. *Ans.*

92. $\frac{2}{5}$ or 60 per cent. of the mixture in S is wine,
$\frac{7}{10}$ or 70 per cent. „ in T „
$\frac{11}{16}$ or $68\frac{3}{4}$ per cent. of the proposed mixture is to be wine; so that we have to find in what proportions mixtures of 60 and 70 per cent. strength are to be combined, in order to produce a third mixture of $68\frac{3}{4}$ per cent. strength. The proportions will be as $70 - 68\frac{3}{4} : 68\frac{3}{4} - 60$, or as $1\frac{1}{4} : 8\frac{3}{4}$, or as 1 : 7.
∴ $\frac{1}{8}$ of the 16 gallon mixture = 2 gall. from S, 14 „ from T } *Ans.*

93.

A obtains $\frac{3\frac{1}{2}}{82\frac{1}{8}}$ or $\frac{28}{657}$ of £1 per £1 of his investment;

B obtains an income $=\frac{3\frac{1}{4}}{81}$ of $\frac{13}{324}$ of £1 per £1 of A's investment, together with $\frac{13}{324}$ of £70; so that $(\frac{28}{657}-\frac{13}{324})$ of A's investment $=(\frac{210}{324}+\frac{5}{6})$ of £1,

or $\frac{9072-8541}{657\times324}$ of A's investment $=\frac{5460+1620}{324\times6}$ of £1,

or $\frac{531}{657}$ of A's investment $=\frac{1}{6}$ of £7080 $=$ £1180;

$\therefore$ £1180 $\times\frac{73}{59}=$ £1460. *Ans.*

94. The fraction is very nearly $=\sqrt{\cdot92}$ or $=\cdot96$. Taking, therefore, $\frac{96}{100}$ or $\frac{24}{25}$ as the value of the fraction, we have $\frac{24}{49}$ of $96=47$, and $\frac{25}{49}$ of $96=49$; hence the fraction is $\frac{47}{49}$. *Ans.*

95. $20s.\times11\frac{2}{3}\div3\frac{1}{2}s.=66\frac{2}{3}$ yards of carpet $\frac{3}{4}$ of a yard wide; hence the area of the floor $=66\frac{2}{3}\times\frac{3}{4}$ $=50$ sq. yds.

Now we have the volume of the room in cubic yards $=50$ times the height in yards, and $=37\frac{1}{2}$ times the breadth, and $=27$ times the length; hence the length is to the breadth as $37\frac{1}{2}:27$, or $25:18$; and hence 50 sq. yds. must be the product of 25 lineal units by 18 such units $=450$ square units; $\therefore$ 1 square unit $=\frac{1}{9}$ of a square yard, and therefore 1 lineal unit $=\frac{1}{3}$ of a yard. Accordingly, the length and breadth of the room are $\frac{25}{3}$ and $\frac{18}{3}$, or $8\frac{1}{3}$ yds. and 6 yds.; also the height being to the length as $27:50$, we have $\frac{27}{50}$ of $8\frac{1}{3}$ $=4\frac{1}{2}$ yds. high.

We can now find the cost of papering the walls:— $(8\frac{1}{3}+6)\times2\times4\frac{1}{2}$ sq. yds. of paper, $\frac{10}{36}$ of a yard wide, $=129\div\frac{10}{36}$ lineal yards at $6d.$, the amount of which in pence is $12\frac{9}{10}\times36\times6=2786\frac{2}{5}d.=$ £11. 12*s.* $2\frac{2}{5}d.$ *Ans.*

96. The interest on £139. 10*s.* is £3. 2*s.*, which is $\frac{622}{27900}$ of $100=\frac{1009}{45}$ or $2\frac{2}{9}$ per cent. Now, interest on

£100 for thrice the time at half the rate is $=\frac{3}{2}$ of $2\frac{2}{3}$ $=3\frac{1}{3}$; hence $\frac{3\frac{1}{3}}{103\frac{1}{3}}$ of £142. 12*s.* $=\frac{1}{31}$ of that sum $=$ £4. 12*s.* *Ans.*

97. $\frac{1}{20}$ of $48=31\frac{1}{5}$ gall. spirits in the first cask,
$\frac{18}{35}$ of $42=\underline{21\frac{3}{5}}$ „ „ „ second,
$52\frac{4}{5}$ gall. spirits in the two,
$90-52\frac{4}{5}=37\frac{1}{5}$ „ water „
$52\frac{4}{5} : 37\frac{1}{5}+20$ as 264 : 286, or as 12 : 13. *Ans.*

98. The contents of the vessels will be successively as follows:—

Contents of the	1st,	spirits	$\frac{1}{8}$	and water	$\frac{7}{8}$,
„	2nd,	„	$\frac{1}{6}+\frac{}{48}$	„	$\frac{35}{48}$,
„	3rd,	„	$\frac{1}{4}+\frac{13}{64}$	„	$\frac{35}{64}$,
„	4th,	„	$\frac{1}{3}+\frac{29}{96}$	„	$\frac{35}{96}$.

Thus in filling up the second vessel we make $\frac{5}{6}$ of it consist of spirits and water in the proportion of 1 : 7; in filling up the third we make $\frac{3}{4}$ of it consist of spirits and water in the proportion of 13 : 35; in filling up the fourth we make $\frac{2}{3}$ of it to consist of spirits and water in the proportion of 29 : 35. Hence, spirits to water in fourth vessel as $\frac{32+29}{96} : \frac{35}{96}$, or as 61 : 35. *Ans.*

99. £3 × (A's no. of cents − B's no.) = $£\frac{4}{5}$;
or A's no. − B's no. $=\frac{4}{15}$.
Again: $£5\frac{1}{4}\times\frac{20}{105}\times$ A's no. of cents = A's 2nd income,
and £4 $\times\frac{21}{84}\times$ B's „ = B's „
hence $4\frac{1}{2}$ times A's no. $-4\frac{1}{3}$ times B's no. $=3\frac{3}{5}$,
but $4\frac{1}{3}$ „ A's $-4\frac{1}{3}$ „ B's $=1\frac{7}{45}$,
∴ $\frac{1}{6}$ of A's $=2\frac{4}{9}$, or A's no. $=14\frac{2}{3}$ cents $=£1466\frac{2}{3}$,
also B's $=14\frac{2}{3}-\frac{4}{15}$ $=14\frac{2}{5}$ cents $=£1440$. } *Ans.*

100. The L.C.M. of 91 and 65, viz. 455, may be chosen as the no. of units in the whole work. Of these A does 30 per hour, B 28, C 25; sum, 83 units per hour, which, if all worked throughout the 7 hours, would amount to 581 units, or 126 more than required; as, therefore, 126 units would employ B for $126 \div 28$ or $4\frac{1}{2}$ hours, that is the time after which he must join the others. *Ans.*

LONDON: PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
AND PARLIAMENT STREET